"All is number." (Pythagoras)

Module 1. Mathematics 101

Unit 1. Digits and Numbers

Figure 1. "Pythagoreans Celebrate the Sunrise" (by Fyodor Bronnikov)

Activity 1. Add the missing forms in the chart and complete the sentences with the words.

Noun (Full)	Noun (Abbreviated)	Noun (Person)	Adjective	Adverb
mathematics	(British) (American)			

1. __________ is a fundamental subject that explores the relationships between numbers, shapes, and patterns.
2. __________ reasoning is essential for solving real-world problems and making informed decisions.
3. I enjoy solving challenging __________ problems; it's a great way to exercise my brain.
4. In my __________ class, we are learning about algebraic equations and their applications.
5. The __________ devoted his life to unraveling the mysteries of prime numbers.
6. The professor explained the concept __________, using rigorous proofs and logical deductions.

Activity 2. Fill in the blanks with the word "digit" or "number" in the correct form (singular or plural).

A symbol that forms part of a (1) __________ is called a (2) __________. For example, the (3) __________ 42.768 has five (4) __________. Ten (5) __________ are used in the decimal system, namely, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to write any (6) __________.

The word (7) __________ also means a finger or a toe. As one learns to count with one's fingers it is not surprising that the word has come to be used for specific (8) __________ the fingers represent.

Activity 3. Match the words with the definitions.

1. a number	a. a symbol that represents a number
2. a number of	b. a sign or word that represents an amount or quantity
3. a numeral	c. to name each one of a series or list of things
4. numerical	d. existing in large numbers
5. numerous	e. basic skills in mathematics
6. numerate	f. expressed as numbers, or consisting of numbers
7. numeracy	g. having basic skills in mathematics
8. to enumerate	h. an unspecified amount, several, or many
9. enumeration	i. the act or process of naming each one of a series or list of things
10. innumerable	j. too many to be counted

Activity 4. Choose the best alternative.

1. A large number / numeral of students demonstrated proficiency in numeracy on the standardized test.
2. A numeral / numerical is a symbol representing a number.
3. Being numerate / numerous requires a solid understanding of arithmetic and mathematical principles.
4. Enumerate / Innumerable the prime numbers between 1 and 20.
5. Enumerate / Numeracy skills are crucial for interpreting statistical data in research studies.
6. Numerate / Innumerable combinations are possible when arranging the coloured tiles in the grid.
7. Numerate / Numerous factors contribute to the complexity of this mathematical problem.
8. The enumerate / enumeration of solutions to the equation revealed multiple valid answers.
9. The number / numeral of apples in the basket is 15.
10. The numerical / numerous solution to the equation is 8.

Activity 5. Study the tables.

Table 1

Cardinal Numbers					
0	zero, nought	10	ten		
1	one	11	eleven		
2	two	12	twelve	20	twenty
3	three	13	thirteen	30	thirty
4	four	14	fourteen	40	forty
5	five	15	fifteen	50	fifty
6	six	16	sixteen	60	sixty
7	seven	17	seventeen	70	seventy
8	eight	18	eighteen	80	eighty
9	nine	19	nineteen	90	ninety

Table 2

Ordinal Numbers					
0^{th}	zeroth (zeroeth)	10^{th}	tenth		
1^{st}	first	11^{th}	eleventh		
2^{nd}	second	12^{th}	twelfth	20^{th}	twentieth
3^{rd}	third	13^{th}	thirteenth	30^{th}	thirtieth
4^{th}	fourth	14^{th}	fourteenth	40^{th}	fortieth
5^{th}	fifth	15^{th}	fifteenth	50^{th}	fiftieth
6^{th}	sixth	16^{th}	sixteenth	60^{th}	sixtieth
7^{th}	seventh	17^{th}	seventeenth	70^{th}	seventieth
8^{th}	eighth	18^{th}	eighteenth	80^{th}	eightieth
9^{th}	ninth	19^{th}	nineteenth	90^{th}	ninetieth

Table 3

In writing, two-word numerals from 21 to 99 are hyphenated (-).	
twenty-one ninety-nine	twenty-first ninety-ninth

Table 4

100	a/one hundred	100^{th}	hundredth
1,000	a/one thousand	$1{,}000^{th}$	thousandth
1,000,000	a/one million	$1{,}000{,}000^{th}$	millionth
1,000,000,000	a/one billion	$1{,}000{,}000{,}000^{th}$	billionth
1,000,000,000,000	a/one trillion	$1{,}000{,}000{,}000{,}000^{th}$	trillionth

Table 5

a single digit a single figure a one-digit number a one-figure number a single-digit number	a multidigit, a multidigit number a multifigure, a multifigure number	
	a double digit a double figure a two-digit number a two-figure number a double-digit number	a triple digit a triple figure a three-digit number a three-figure number a triple-figure number
1, 2, 3	*12, 34, 56*	*123, 456, 789*

Table 6

987 nine hundred and eighty-seven (BrE)
987 nine hundred eighty-seven (AmE)
987 nine eighty-seven (AmE, informal)

Table 7

1100 one thousand one hundred OR eleven hundred
9900 nine thousand nine hundred OR ninety-nine hundred

Table 8

In high numbers, the words "hundred", "thousand" etc. do not take an "s".	"s" is used when referring to approximate quantities and is often followed by "of".
24 two dozen	dozens of
300 three hundred	hundreds of
4,000 four thousand	thousands of
5,000,000 five million	millions of
6,000,000,000 six billion	billions of
7,000,000,000,000 six trillion	trillions of

Table 9

Negative Numbers **Positive Numbers**

–2	–1	0	1	2
minus two	minus one	zero	one	two
negative two	negative one	nought		

Table 10

	Whole Number Part				Decimal Point	**Fractional Part**		
Places	Thousands	Hundreds	Tens	Ones Units		Tenths	Hundredths	Thousandths
Place Values	1	2	3	4	.	5	6	7
	1,234.567 one thousand two hundred thirty-four point five six seven							

Table 11

In English, integers (whole numbers) are written with commas (,) and decimals with dots (.).
123,456 one hundred twenty-three thousand four hundred fifty-six **456.789** four hundred fifty-six POINT seven eight nine

Table 12

Fractions	
Fractions **Vulgar Fractions** **Common Fractions**	**Decimals** **Decimal Fractions**
$\frac{1}{2}$ a/one half, one over two $\frac{4}{2}$ four halves, four over two $\frac{1}{4}$ a/one quarter, a/one fourth, one over four $\frac{2}{4}$ two quarters, two fourths, two over four $5\frac{3}{7}$ five and three sevenths, five and three over seven	**0.5** point five, zero point five, nought point five **1.5** one point five **11.55** eleven point five five **123.456** one hundred twenty-three point four five six

Table 13

$\frac{numerator}{denominator}$			
a proper fraction	$\frac{3}{4}$	**an improper fraction**	$\frac{4}{3}$
a complex fraction	$\frac{3/5}{4/7}$		
a mixed fraction **a mixed number**	$3\frac{1}{2}$	**a mixed decimal** **a decimal mixed number**	2.38

Table 14

Percentages	Fractions, Decimals, Percentages
1% one per cent (percent) **10%** ten per cent **12.34%** twelve point three four per cent	$\frac{\mathbf{9}}{\mathbf{10}}$ nine tenths, nine over ten **0.9** (zero/nought) point nine **90%** ninety per cent

Table 15

Ratios	Proportions
1:3 one to three	**A:B = C:D** A is to B as C is to D $\frac{\boldsymbol{A}}{\boldsymbol{B}} = \frac{\boldsymbol{C}}{\boldsymbol{D}}$ A is to B as C is to D

Activity 6. Read aloud the numbers.

1) 0
2) 31st
3) 42nd
4) 53rd
5) 64th
6) 75th
7) 86th
8) 97th
9) 300
10) 400
11) 500
12) 6,000
13) 7,000
14) 8,000
15) 132
16) 132nd
17) 243
18) 243rd
19) 354
20) 354th
21) 4,765
22) 4,765th
23) 5,876
24) 5,876th
25) 6,987
26) 6,987th
27) 2,000,000,000
28) $\frac{3}{2}$
29) $\frac{1}{3}$
30) $\frac{2}{4}$
31) $\frac{3}{5}$
32) $3\frac{5}{7}$
33) $4\frac{6}{8}$
34) $5\frac{7}{9}$
35) 0.312
36) 3.1
37) 4.21
38) 55.55
39) 654.456
40) 789.9
41) 2%
42) 31%
43) 50%
44) 64.5%
45) 70.79%
46) 87.987%

Activity 7. Label each number an integer or a decimal. Read them out loud.

1) 1,946
2) 7,498
3) 34,209
4) 52,867
5) 76,804
6) 85,432
7) 296,500
8) 6.409
9) 498,903
10) 535,600
11) 124.575
12) 8.702
13) 816,492
14) 37.897
15) 26.936
16) 231.973

Activity 8. Make a list of ten multidigit numbers: cardinal, ordinal, integers, fractions, decimals, percentages. Without showing the list, read it aloud for your partner to write down symbolically. Check if the numbers are correct. Swap roles.

Activity 9. Name the place of each digit within the numbers.

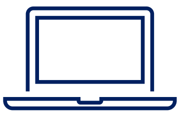
1) 50
2) 501
3) 519
4) 950.1
5) 95.01
6) 9.015
7) 195.91
8) 915.159

Activity 10. Give examples to illustrate the notions.

1. a digit
2. a cardinal number
3. an ordinal number
4. a single-digit number
5. a multidigit number
6. a double figure
7. a triple figure
8. a place
9. a fraction
10. a common fraction
11. a decimal
12. a percentage
13. a ratio

Activity 11. Replace one word or phrase in each sentence with the pronoun “one”.

1. If a person enjoys solving puzzles, the person might find satisfaction in mathematical challenges.
2. If a student encounters difficulties, the student can seek assistance from the teacher.
3. If you find a red apple, please give me the red apple.
4. If you're looking for a good book, I recommend this book.
5. In a group discussion, it's important to consider everyone's ideas and choose the best idea.
6. In times of uncertainty, it's natural for a person to seek guidance from trusted sources.
7. It's always good to have a backup plan; no one wants to be caught without a backup plan.
8. Learning a new language can be challenging, but with dedication, a person can become fluent in a new language.
9. When attending a party, it's customary to bring a gift for the host, even if it's a small gift.
10. When faced with a difficult decision, trust your instincts; they can guide you to the right decision.



Activity 12. Match the definitions of the word “figure” (a–h) with the sentences (1–8).

a. a geometric shape
b. a human shape
c. a particular amount expressed as a number, especially a statistic
d. a person, especially an important one
e. an illustration or diagram in a text
f. another name for a digit
g. to calculate or compute
h. to think or guess

1. How can you **figure** the volume of cylinder?
2. If we can **figure** roughly how much it will cost, we can decide what to do.
3. It would be difficult to carry out an eight-**figure** calculation without a computer.
4. Leonard Euler is a distinguished **figure** in mathematics.
5. Similar **figures** have many geometric properties in common.
6. The illustration of the model is shown in **Figure** 2.
7. The latest unemployment **figures** are discouraging.
8. There before him stood a tall **figure** in black.

Activity 13. Research the subject "Establishment of Zero as a Number". Report your findings with a multimedia presentation.

Checklist for a multimedia presentation:

1. Remember that the visuals (presentation) should only be used to support the presenter's message.
2. The key word to prepare an effective presentation is **MINIMUM**:
   - make the necessary **minimum** number of slides;
   - reduce text to a **minimum** (proper names, dates, figures, graphs etc.), try not to write complete sentences;
   - use bullet points to structure information (as in Point 1);
   - keep design and content simple.
3. Remember the rule of six:
   - a maximum of six lines per slide,
   - a maximum of six words per line.
4. Check whether the visual really shows what you are saying.
5. Make sure your audience can read the visual (font size and colours).
6. Find effective headlines.
7. The last slide REFERENCES shows the list of sources you used to prepare your report.
8. When reporting speak to the audience and don't read the slides.

Unit 2. Numeracy

Activity 14. Complete the sentences with the words in the box.

calculate / count / equals / measure / solve

1. If something ___________ a particular number or amount, it is the same as that amount or the equivalent of that amount.
2. If you ___________ a number or amount, you discover it from information that you already have, by using arithmetic, mathematics, or a special machine.
3. If you ___________ a quantity that can be expressed in numbers, such as the length of something, you discover it using a particular instrument or device, for example a ruler.
4. If you ___________ a problem or a question, you find a solution or an answer to it.
5. When you ___________, you say all the numbers one after another up to a particular number.

Activity 15. Choose the best alternative.

1. An equation / inequality like $8 < 12$ indicates that 8 is less than 12.
2. Equality / Inequality in education is a fundamental principle that promotes fairness.
3. I used a calculate / calculator to determine the average of the test scores.
4. I will solve / solution the equation $3x + 7 = 22$ to find the value of x.
5. Some items, like grains of sand, are practically countable / uncountable due to their vast quantity.
6. The calculable / incalculable number of grains of sand on the beach makes it impossible to count precisely.
7. The calculate / calculation of the perimeter involves adding the lengths of all sides.
8. The counting / countless exercise revealed that there are ten pencils on the desk.
9. The discount on the sale items is easily calculable / incalculable using a percentage.
10. The equal / unequal distribution of resources can lead to social disparities.

Activity 16. Read the passage. In pairs, discuss the questions in the box. When discussing you can use the connective words given in Appendix I. Cohesive Devices.

Numeracy is defined as the ability to understand and use maths in daily life, at home, work, or school; it doesn't mean complex skills, like algebra, it means being confident enough to use basic maths in real-life situations. Numeracy is not always taught in the classroom: it means having the confidence and skills to use maths to solve problems in everyday life. Numeracy is as important as literacy — it's sometimes called "mathematical literacy" — and we need both to get on in life. Numeracy means understanding how maths is used in the real world and being able to apply it to make the best possible decisions. It's as much about thinking and reasoning as about "doing sums". It means being able to: interpret data, charts, and diagrams; process information; solve problems; check answers; understand and explain solutions; make decisions based on logical thinking and reasoning.

(from National Numeracy)

1. What is numeracy?
2. What does it mean to be numerate? innumerate?
3. Is numeracy a basic or special skill? Why?
4. What are the benefits of numeracy?
5. What are the challenges of innumeracy?

Activity 17. Match the synonymous items in the two columns.

1. an area	a. to use
2. an upfront response	b. to increase greatly
3. diverse applications	c. to establish
4. the impact of	d. to be front and centre
5. to be efficient	e. to be essential for
6. to be integral to	f. to be effective
7. to be pervasive	g. the effect of
8. to determine	h. different uses
9. to employ	i. a field
10. to rise exponentially	j. a direct answer

Activity 18. Read the article to enumerate the applications of mathematics described in the text. Extend the list with your ideas. For that, use Appendix I. Cohesive Devices.

"When will you ever need to use maths in real life?" It's the question asked perennially in maths classrooms up and down the country and indeed around the world. The answer is easy — perhaps we should be responding to the question with the counter, "When will you not?"

The pandemic has provided some very upfront answers. For the last few years, we have been hearing regularly about the potential for cases to rise exponentially. News programmes carried regular features on the reproduction number, R. Others reported, in vain, that we might nearly have reached elusive mathematically defined herd immunity thresholds.

We relied on mathematical models, not only to understand the current situation but to predict what might happen in the future, from the impact of mitigations to the effectiveness of vaccines. We used maths to determine the most efficient order to deliver jabs during the vaccine rollout and to plan the roadmap out of lockdown in early 2021. Maths was front and centre much of the time.

Even outside times of crisis we see maths in the newspaper headlines every day. We use it to establish whether our politicians are telling the truth about unemployment. Maths allows us to monitor exchange rates during currency crashes. It is invaluable to opinion pollsters determining the popularity of our political parties and to fact-checkers holding politicians to account.

Away from the front-page headlines, maths is the language of science. It appears everywhere from physics to engineering and chemistry — aiding us in understanding the origins of the universe and building bridges that won't collapse in the wind. Perhaps a little more surprisingly, maths is also increasingly integral to biology. Scientists in my own specialist area of mathematical biology are helping to develop treatments for diseases and to answer the question of how the leopard got its spots.

Beyond the academy, we are increasingly employing maths in sport to enhance the performance of our top athletes. We use it in the movies to create computer-generated images of scenes that couldn't exist in reality. More mundanely, we are frequently using maths in our everyday lives when we go shopping or when we follow a recipe, when we tell the time or when we budget for the future. Much of the time we do it without even realising it.

Certainly, much of the maths we learn early on in school we use directly in our everyday lives. Other topics that we might have learnt later, or perhaps we never got around to, are essential for the functioning of modern society even if we don't often see their use directly.

There are of course bits of maths (particularly pure maths) for which it is harder to imagine a direct use. But isn't this true of every subject? Should we hold geography, for example, to the same exacting standards of utility we expect of maths? I don't remember the last time I put my hard-won knowledge of oxbow lakes to use. Similarly, in chemistry, when was the last time you needed to write down the chemical reaction diagram depicting esterification? Probably not very recently.

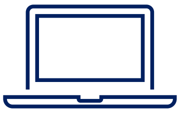
This is not to denigrate these subjects, but to point out that this is not a maths-specific issue. Perhaps maths suffers more because it is harder to visualise the direct application of an algebraic equation than it is to picture the flow of water in a river, for example. We can all remember sitting by a river watching the water flow past, but fewer of us, I would suggest, can imagine laying down our picnic blanket on the complex plane of an Argand diagram.

By necessity, maths tends to deal in generalities and therefore abstractions from reality. But, at least in part, it is the generality — the abstractness — which makes mathematics so pervasive.

At university, I teach students that a single abstract equation can describe the spread of heat through your radiator, the diffusion of a drop of food colouring in a glass of water and the random dispersion of cells on a petri dish. With such a diverse range of applications, you can start to see how powerful it is to study a seemingly abstract and lifeless equation for the deep insights it can provide about ostensibly unrelated systems.

It was not for nothing that philosopher Eugene Wigner wrote of "the unreasonable effectiveness of mathematics" for describing the natural world. Many simple mathematical ideas come up over and over again in different areas. The "normal distribution" — or bell curve — for example, can be used to describe people's IQs as well as their heights and has hundreds of other applications too. The problem mathematics faces may be that it has too many applications.

(by Kit Yates, from The Independent, 2022)



Activity 19. Watch the video "Why Do People Get So Anxious About Math?" to choose the best answer to the questions. Does being bad at mathematics render one unintelligent? Why? Watch the video again and make a note of all the myths and stereotypes associated with mathematics. Disprove them. When expressing disagreement, use Appendix I. Cohesive Devices. Disagreeing.

https://disk.yandex.ru/i/iqnZm-k2Yh-Hrg

1. What happens to working memory when someone experiences math anxiety?

A. It increases and helps solve problems faster
B. It decreases because worry uses it up
C. It stays the same but becomes more efficient
D. It completely stops functioning during tests

2. Why does math anxiety seem to be more common than anxiety in other subjects?
A. Math is naturally harder than all other subjects
B. Only intelligent people can experience math anxiety
C. How parents and teachers present math to children influences their anxiety levels
D. Students spend more time studying math than other subjects

3. What do relaxation techniques and writing down worries have in common as strategies for math anxiety?
A. They both require a teacher's supervision
B. They both help free up working memory to focus on math
C. They both take several hours to be effective
D. They both work only for elementary school students

4. What does the "growth mindset" principle suggest about mathematical ability?
A. Only children can develop their math skills
B. Math skills are determined at birth and cannot change
C. The brain areas for math can develop and improve over time
D. Growth mindset only works for people who are already good at math

5. What advice does the text give to teachers and parents of young children?
A. Focus on speed and quick problem-solving
B. Separate boys and girls during math lessons
C. Be playful with math and allow time for children to work through answers
D. Tell children that math is challenging so they take it seriously

Activity 20. Label the points related to mathematical anxiety as either a cause, symptom, effect, or solution.

1) avoiding everyday situations involving maths at work or at home, like helping children with homework;
2) being in pressured situations, such as fearing being judged on how quickly you can produce an answer, or sitting an exam;
3) blocking any motivation to practise in order to learn and progress;
4) creating or amplifying a belief that maths ability is "fixed" and cannot be improved;
5) easing into it, working at your own pace, without the pressure to master a problem straight away;

6) feeling flustered, panicked or stressed, experiencing sweating and nausea, having increased heart rate;
7) having cultural bias, for example implications from opinions in the media and popular culture that because of background or gender someone is likely to have lower ability in maths;
8) having reduced performance in some situations and tests;
9) having specific negative past experiences, for example having felt humiliated for getting something wrong while in school;
10) leaving the individual caught in a cycle of anxiety;
11) making the time, just ten minutes here and there, to give sums a go, ideally somewhere relaxed, so it doesn't feel like a test environment;
12) overcoming maths myths;
13) preventing people from applying for courses, jobs, and promotions;
14) recognising the emotion and that it won't always be this way, i.e. that this is the way that you feel now, but not forever;
15) remembering that the ability to be good at maths isn't something we are born with; it can change over time, and we can all be good with numbers;
16) setting achievable goals, which feel reachable;
17) struggling to concentrate on a calculation;
18) talking it through, looking online, or asking a colleague or friend what they would do.

Activity 21. In groups, discuss the points.

1. Reflect on personal experiences with mathematical anxiety. Can you relate to situations where confidence in mathematical ability varies depending on the context? Share your own experiences if applicable.
2. Discuss the phenomenon of imposter syndrome in the context of mathematical anxiety. How does the fear of not measuring up to others impact one's capacity to think logically, especially in public spaces?
3. Explore the potential impact of mathematical anxiety on life decisions and career choices. How might a person's avoidance of mathematical situations influence their professional path, and what are the implications for educational and career opportunities?
4. Investigate the cyclic nature of mathematical anxiety and its potential causes. How do negative experiences with mathematics contribute to anxiety, and what factors may be under the control of educators to mitigate these issues?
5. Consider long-term solutions to address mathematical anxiety, such as focused professional development for teachers and changing the prevailing "can't do" culture. Discuss the feasibility and potential impact of these solutions on creating a more positive learning environment.

6. Explore the concept of mathematical resilience. How can fostering a "can do" attitude and a growth mindset contribute to overcoming mathematical anxiety? Share strategies that promote a positive idea of resilience in mathematics.
7. Examine the gender differences in the prevalence of mathematical anxiety, particularly its higher occurrence in girls. How do social and historical factors contribute to these differences, and what steps can be taken to encourage interest in maths among all students, regardless of gender?
8. Discuss the function of role models, particularly in the context of female mathematicians. How can role models contribute to breaking down stereotypes and encouraging a positive attitude toward mathematics, especially among girls?

Activity 22. Write a paragraph on the role that mathematics plays in day-to-day life. Follow the plan:

1. Start by clearly stating the main idea. (e.g., "*Mathematics plays a crucial and often unseen role in our daily lives*.").
2. Provide 2–3 concrete examples from everyday life. (e.g., *budgeting, cooking, shopping, time management, sports*).
3. Briefly explain how math is used. (e.g., "*We use ratios and proportions when following a recipe*.").
4. Use linking words to make your writing smooth (e.g., *For example, Furthermore, Another area is, In conclusion*).
5. End with a sentence that summarizes your point and shows why it's important.

Unit 3. Defining Mathematics

Activity 23. Complete the spaces with the correct form of the words from the box.

branch / calculate / count / mathematician / mathematics / measure / problem

(1) ___________ is arguably the oldest of the sciences. It began with man's need to (2) ___________ objects and to (3) ___________ distances. A (4) ___________ uses numbers and signs to (5) ___________ fixed quantities or to compute variable quantities.

Mathematics is known as the most exact of all the sciences since the proper use of its methods can provide only one correct answer to a specific (6) ___________. It is the language used by all the other sciences. It is the basis for precision in such (7) ___________ as astronomy, chemistry, and physics.

Activity 24. Choose the best alternative.

1. The equality / equation 2x + 5 = 11 can be solved to find the value of x.
2. The immeasurable / measurable vastness of the universe captivates our imagination.
3. The impact of the discovery was immeasurably / measurably significant for scientific progress.
4. The measure / measurement of the rectangle's sides indicated it was a square.
5. The mystery remained solved / unsolved, leaving the investigators puzzled.
6. The number of stars in the sky seems countable / countless on a clear night.
7. The solve / solution to the problem is 15, as confirmed by my calculations.
8. The solved / unsolved puzzle revealed a beautiful image of a rainbow.
9. The weights of the two boxes are equal / equate.
10. Using a computer program, it is easy to calculate / count the square root of any given number.

Activity 25. In pairs, discuss the questions.

1. How would you define mathematics?
2. Why do you think some people state that it is hard to define mathematics?
3. Which descriptor(s) and why would you use to define mathematics: (a) an innate ability; (b) a human construct; (c) a language; (d) a natural phenomenon; (e) a philosophy; (f) a religion; (g) a science; (h) a tool; (i) an art; (j) the truth?

Activity 26. Read the article to collect all the definitions of mathematics given in the text. Classify them as either formal or informal.

Believing myself to be a non-mathematician, my dormant interest in maths has been awakened by the lively discussions going on around me every day, and the environment in which I am steeped has prompted me to ask the question "What is mathematics?" My search for the answer has unearthed some interesting ideas which I'd like to share.

In her blog on mathematics and ethics Lucy Rycroft-Smith says that mathematics is "the language of pattern, measurement and logical rules". Now the idea of mathematics being a language resonates with me, evoking the memory of a statement that mathematicians the world over can communicate and understand each other through this "metalanguage par excellence".

In a blog on the role of paradox in mathematics, Vinay Kathotia stated that "mathematics is the art of interpreting, quantifying, and working with error and uncertainty". This gave me pause — I had never thought of maths as an art before. I have marvelled at the art produced by mathematics — for instance, the boundless beauty of fractals or the complexity of my son's computer-designed creations — but to think of mathematics itself as an art, and especially as an art which deals with error and uncertainty was beyond my imagining. Hadn't I always been taught that maths was certain, more like a science based on logic and facts, and that errors were wrong, resulting in red crosses all over my exercise book?

Moving on to a computer search the inevitable Wikipedia provided the less-than-helpful (although no doubt accurate) "Mathematics has no generally accepted definition", before looking at a variety of suggested definitions ranging from Aristotle ("the science of quantity"), through abstract and philosophical definitions ("symbolic logic"; "carrying out mental constructions"; "the examination of the properties and interactions of idealized objects"), to humorous or even poetical definitions ("the art of giving the same name to different things" — there's that word art again).

A more thorough online search however led me to these wonderful words by Dr Liaqat Khan, Professor in Mathematics at Quaid-i-Azam University, Islamabad: "Mathematics is concerned with using imagination, intuition and reasoning to find new ideas and to solve puzzling problems". Whilst his use of the qualifier "concerned with" means this is not a definition per se, the overarching vision he expresses is inspiring and excitingly inclusive. If we accept this statement, then we are all born mathematicians; we are inherently mathematical beings whose defining human characteristics (as opposed to those we share with other animals) are those which also make us mathematical, and I can no longer claim to be a non-mathematician.

What is more, no-one should be viewed by others as a non-mathematician, especially by those whose aim is to teach mathematics. If mathematics is a language, an art, a science, a philosophy, as well as innate, and if it embraces those very human traits of error and uncertainty, then learning mathematics should be delightful for all of the budding linguists, artists, scientists, philosophers and yes, even career mathematicians in each classroom. The challenge, of course, is finding ways of nurturing this delight — the imagination and intuition as well as the reasoning — within the constraints of a set curriculum.

(by Lynn Fortin, from Cambridge Mathematics, 2018)

Activity 27. Complete the chart with as many definitions of mathematics you can find using additional resources. Include your critical evaluation of each definition in the Notes column. Select or, if needed, formulate the optimum definition, and explain your rationale.

№	Author	Bio	Definition	Notes
1	Aristotle	Ancient Greek philosopher (4^{th} century B.C.E.)	the science of quantity	*incomplete*
…				

Activity 28. Do you believe in the idea of a mathematical gene, that a mathematical ability is genetic? Dissect the weight that nature (genetics) and nurture (upbringing and education) carry in the process of developing mathematical knowledge and skills. Debate in groups.

Activity 29. Choose one quote and comment on it in writing.

1. "If I were again beginning my studies, I would follow the advice of Plato and start with mathematics." (Galileo Galilei)
2. "Life is good for only two things, discovering mathematics and teaching mathematics." (Siméon Denis Poisson)

Unit 4. Language of Mathematics

Activity 30. Complete the sentences with the phrases by combining the items in the two columns. Use the initial letters as clues. Some forms need changing.

1) achieve 2) carry out 3) conduct 4) do 5) find 6) find the answer to 7) make 8) perform 9) produce 10) reach 11) satisfy 12) solve 13) undertake 14) work out	a) calculation b) equation c) operation d) problem e) research f) solution

1. After careful consideration, we were able to r__________ a satisfactory s__________.
2. After hours of effort, I finally a__________ a s__________ to the complex equation.
3. Can you w__________ o__________ the e__________ for the area of the triangle?
4. He is skilled at helping students s__________ complex math p__________.
5. I always enjoy d__________ math p__________ in my spare time.
6. I need to d__________ a c__________ to determine the total cost.
7. It took me a while to s__________ the e__________, but I finally got the answer.
8. It took some time, but I finally f__________ the a__________ to the p__________.
9. I finally managed to w__________ o__________ a s__________ to the problem.
10. Let's m__________ a c__________ to figure out the average.
11. Let's work together to f__________ a s__________ to this challenging problem.
12. Mathematicians c__________ r__________ to push the boundaries of mathematical knowledge.
13. Scientists often d__________ r__________ to explore new mathematical theories.
14. She decided to u__________ r__________ to f__________ innovative s__________.
15. The scientist will c__________ o__________ o__________ on the data to analyze the results accurately.
16. The team collaborated to p__________ a s__________ that satisfies all requirements.
17. The values I found s__________ the e__________ perfectly.
18. To find the area of the rectangle, we need to p__________ the o__________ of multiplying its length by its width.

Activity 31. In pairs, discuss the questions.

1. What is meant by the language of mathematics?
2. Is mathematical language international? Why?
3. Is mathematical language universal? Why?
4. Do you find the language of mathematics to be easy or difficult? Why?

Activity 32. Label the concepts as algebraic or geometric.

1. circumference	5. imaginary number	9. perimeter	13. product
2. cone	6. multiplication	10. pi	14. quarter
3. cube	7. obelus	11. polygon	15. side
4. foot	8. parallelogram	12. problem	16. triangle

Activity 33. Read the article and match the headings (a–f) to the passages (1–6).

(a) International Rules
(b) Language as a Teaching Tool
(c) The Argument Against Math as a Language
(d) Vocabulary, Grammar, and Syntax in Mathematics
(e) What Is a Language?
(f) Why Mathematics Is a Language

(1) ____

Mathematics is called the language of science. Italian astronomer and physicist Galileo Galilei is attributed with the quote, "Mathematics is the language in which God has written the universe." Most likely this quote is a summary of his statement in Opere Il Saggiatore:

"[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word."

Yet, is mathematics truly a language, like English or Chinese? To answer the question, it helps to know what language is and how the vocabulary and grammar of mathematics are used to construct sentences.

In order to be considered a language, a system of communication must have vocabulary, grammar, syntax, and people who use and understand it.

Mathematics meets this definition of a language. Linguists who don't consider math a language cite its use as a written rather than spoken form of communication.

Math is a universal language. The symbols and organization to form equations are the same in every country of the world.

(2) ____

There are multiple definitions of "language." A language may be a system of words or codes used within a discipline. Language may refer to a system of communication using symbols or sounds. Linguist Noam Chomsky defined language as a set of sentences constructed using a finite set of elements. Some linguists believe language should be able to represent events and abstract concepts.

Whichever definition is used, a language contains the following components:

1. There must be a vocabulary of words or symbols.

2. Meaning must be attached to the words or symbols.

3. A language employs grammar, which is a set of rules that outline how vocabulary is used.

4. A syntax organizes symbols into linear structures or propositions.

5. A narrative or discourse consists of strings of syntactic propositions.

6. There must be (or have been) a group of people who use and understand the symbols.

Mathematics meets all of these requirements. The symbols, their meanings, syntax, and grammar are the same throughout the world. Mathematicians, scientists, and others use math to communicate concepts. Mathematics describes itself (a field called meta-mathematics), real-world phenomena, and abstract concepts.

(3) ____

The vocabulary of math draws from many different alphabets and includes symbols unique to math. A mathematical equation may be stated in words to form a sentence that has a noun and a verb, just like a sentence in a spoken language. For example:

$3 + 5 = 8$

could be stated as "Three added to five equals eight."

Breaking this down, nouns in math include:

1. Arabic numerals (0, 5, 123.7)
2. Fractions (1⁄4, 5⁄9, 2 1⁄3)
3. Variables (a, b, c, x, y, z)
4. Expressions (3x, x2, 4 + x)
5. Diagrams or visual elements (circle, angle, triangle, tensor, matrix)
6. Infinity (∞)
7. Pi (π)
8. Imaginary numbers (i, -i)
9. The speed of light (c)

Verbs include symbols including:

1. Equalities or inequalities (=, <, >)
2. Actions such as addition, subtraction, multiplication, and division (+, -, x or *, ÷ or /)
3. Other operations (sin, cos, tan, sec)

If you try to perform a sentence diagram on a mathematical sentence, you'll find infinitives, conjunctions, adjectives, etc. As in other languages, the role played by a symbol depends on its context.

(4) ____

Mathematics grammar and syntax, like vocabulary, are international. No matter what country you're from or what language you speak, the structure of the mathematical language is the same.

Formulas are read from left to right.

The Latin alphabet is used for parameters and variables. To some extent, the Greek alphabet is also used. Integers are usually drawn from i, j, k, l, m, n. Real numbers are represented by a, b, c, α, β, γ. Complex numbers are indicated by w and z. Unknowns are x, y, z. Names of functions are usually f, g, h.

The Greek alphabet is used to represent specific concepts. For example, λ is used to indicate wavelength and ρ means density.

Parentheses and brackets indicate the order in which the symbols interact.

The way functions, integrals, and derivatives are phrased is uniform.

(5) ____

Understanding how mathematical sentences work is helpful when teaching or learning math. Students often find numbers and symbols intimidating, so putting an equation into a familiar language makes the subject more approachable. Basically, it's like translating a foreign language into a known one.

While students typically dislike word problems, extracting the nouns, verbs, and modifiers from a spoken/written language and translating them into a mathematical equation is a valuable skill to have. Word problems improve comprehension and increase problem-solving skills.

Because mathematics is the same all over the world, math can act as a universal language. A phrase or formula has the same meaning, regardless of another language that accompanies it. In this way, math helps people learn and communicate, even if other communication barriers exist.

(6) ____

Not everyone agrees that mathematics is a language. Some definitions of "language" describe it as a spoken form of communication. Mathematics is a written form of communication. While it may be easy to read a simple addition statement aloud (e.g., 1 + 1 = 2), it's much harder to read other equations aloud (e.g., Maxwell's equations). Also, the spoken statements would be rendered in the speaker's native language, not a universal tongue.

However, sign language would also be disqualified based on this criterion. Most linguists accept sign language as a true language. There are a handful of dead languages that no one alive knows how to pronounce or even read anymore.

A strong case for mathematics as a language is that modern elementary-high school curricula uses techniques from language education for teaching mathematics. Educational psychologist Paul Riccomini and colleagues wrote that students learning mathematics require "a robust vocabulary knowledge base; flexibility; fluency and proficiency with numbers, symbols, words, and diagrams; and comprehension skills."

(by Anne Marie Helmenstine, from ThoughtCo, 2019)

Activity 34. Study Table 16. Relate and specify its levels to the stages of education.

Table 16. OECD Numeracy Framework

<table>
<tr><td>Below Level 1</td><td>At this level individuals must be able to carry out simple processes such as:
counting and sorting;
performing basic arithmetic operations with whole numbers and money;
recognising common spatial representations;
other familiar contexts where the mathematical content is clear with little or no text.</td></tr>
<tr><td>Level 1</td><td>At this level individuals must be able to carry out simple, one-step mathematical processes where the mathematical content is clear with little text, such as the following:
counting and sorting;
performing basic arithmetic operations;
understanding simple percentages;
locating and identifying simple, common graphical, or spatial representations.</td></tr>
<tr><td>Level 2</td><td>At this level individuals must be able to identify and act on mathematical information embedded in a range of common contexts where the mathematical content is fairly clear or visual. Tasks tend to require the application of two or more steps, such as the following:
processes involving calculation with whole numbers and common decimals;
percentages and fractions;
simple measurement and spatial representation;
estimation;
interpretation of relatively simple data and statistics in texts, tables, and graphs.</td></tr>
<tr><td>Level 3</td><td>At this level individuals must be able to understand mathematical information that may be less clear, embedded in contexts that are not always familiar and represented in more complex ways. Tasks require several steps and may involve the choice of problem-solving strategies, such as the following:
application of number sense and spatial sense;
recognising and working with mathematical relationships, patterns, and proportions expressed in verbal or numerical form;
interpretation and basic analysis of data and statistics in texts, tables, and graphs.</td></tr>
<tr><td>Level 4</td><td>At this level individuals must be able to understand a broad range of mathematical information that may be complex, abstract, or embedded in unfamiliar contexts.</td></tr>
</table>

	These tasks involve undertaking multiple steps and choosing relevant problem-solving strategies, such as the following: • analysis and more complex reasoning with quantities and data; • statistics and probability; • spatial relationships; • change, proportions and formulas; • understanding arguments and communicating explanations for answers.
Level 5	At this level individuals must be able to understand complex representations, abstract and formal mathematical and statistical ideas, possibly embedded in more complex forms. This includes the following: • integrating multiple types of mathematical information where considerable interpretation is required; • coming to well-reasoned conclusions; • working with mathematical arguments and models; • justifying, evaluating, and critically reflecting on answers.

Activity 35. In groups, construct a general model of mathematical competency, both knowledge-wise and skill-wise. Include mathematical language competence from the text in Activity 33. Share your ideas with other groups using a multimedia presentation.

Activity 36. Speak on the subject "Mathematics as a Universal Language and the Language of the Universe". As separate paragraphs, include the introduction with your thesis statement, the body with your arguments, and the conclusion.

Unit 5. Branches of Mathematics

Activity 37. Use one and the same word to complete all the sentences. Define the meaning of the word in each sentence.

1. A __________ of mathematics known as geometry focuses on the properties and relationships of shapes.
2. He chose to __________ out in his career, exploring opportunities in different fields.
3. The bank opened a new __________ in the city to expand its services.
4. The company decided to __________ into international markets to expand its business.
5. The detective carefully examined every __________ of the investigation to solve the complex case.
6. The family tree showed how each __________ represented a different generation.
7. The oak tree had a thick __________, providing shade on a sunny day.
8. The river split into a smaller __________, creating a picturesque scene in the forest.

Activity 38. Complete the chart with the names of scientific fields.

<table>
<tr><td colspan="5">anthropology / astronomy / biology (life science) / chemistry / computer science / Earth science / economics / engineering / history / information science / law / linguistics / logic / mathematics / medicine / pedagogy / physics / political science / psychology / sociology</td></tr>
<tr><th colspan="4">Fundamental Sciences</th><th rowspan="2">Applied Sciences</th></tr>
<tr><th colspan="2">Natural Sciences</th><th>Social Sciences</th><th>Formal Sciences</th></tr>
<tr><th>Physical Science</th><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td></td></tr>
</table>

Activity 39. In pairs, discuss the questions.

1. Is mathematics a natural science, a social science, or a fundamental science? Why?
2. What subdisciplines constitute mathematics?
3. What areas of mathematics do you lean towards most? least? Why?
4. What areas of mathematics do you find most challenging? least challenging? Why?

Activity 40. Read the article to differentiate between pure mathematics and applied mathematics, conceptually and historically.

The study of abstract mathematical systems and structures, without necessarily having practical applications in mind, is called pure mathematics. It has various branches, including abstract algebra, geometry, number theory, calculus, topology, and the topics derived from them. The study and use of the mathematical techniques to solve practical problems is called applied mathematics. The field has various branches including statistics, probability, mechanics, mathematical physics.

The distinction between pure mathematics and applied mathematics might not be sharp. For example, Euclidean geometry could be analyzed as an abstract study of the relationships between lines, points, and geometric shapes based on the foundations of Euclid's postulates, or could, at the same time, be viewed as a study of results that could potentially (and, in fact, has proved to be) useful to architects, surveyors, engineers, and scientists. Or, the general study of vectors and vector spaces can be viewed as either an abstract study or a practical one if one later has in mind to use this theory to analyze force diagrams in mechanics.

Although much of the mathematics developed in the time of antiquity was clearly motivated by practical concerns, the development of mathematics for its own sake was nonetheless of interest to early scholars. For instance, Babylonian tablets from ca. 1600 B.C.E. list large Pythagorean triples that could have no practical use. Greek mathematicians of around 400 B.C.E. began to seek rigour, proof, and justification in their mathematical thinking, and ca. 300 B.C.E. Euclid produced his logically rigorous treatise "The Elements", summarizing all mathematical knowledge known at his time. The unique organization of ideas presented in his work became the key feature of the piece. That, in itself, was seen as an analysis of logical thinking, one that became the paradigm of all mathematical and scientific thinking for the two millennia that followed.

During the 19th century, mathematicians began to search for unifying ideas between distinct branches of algebra and geometry. The general study of structures and operations on them led to the development of abstract algebra, for instance. The development of paradoxes in set theory and in the foundations of calculus forced scholars to seek greater levels of rigour and abstraction. Even the nature of logical reasoning itself was examined as an attempt to understand and resolve fundamental paradoxes. The need for abstract analysis and synthesis was recognized as important, and dichotomy between applied and pure mathematics became more apparent.

(from Encyclopaedia Britannica)

Activity 41. Identify the branches of mathematics based on the descriptions.

algebra / analysis / analytic geometry / applied mathematics / arithmetic / calculus / differential calculus / Euclidean geometry / geometry / group theory / integral calculus / logic / non-Euclidean geometry / number theory / probability theory / pure mathematics / set theory / statistics / topology / trigonometry

1. The branch of algebra that deals with mathematical groups.
2. The branch of calculus concerned with the determination of integrals and their application to the solution of differential equations, the determination of areas and volumes, etc.
3. The branch of calculus concerned with the study, evaluation, and use of derivatives and differentials.
4. The branch of geometry describing the properties of a figure that are unaffected by continuous distortion, such as stretching or knotting.
5. The branch of geometry that uses algebraic notation and analysis to locate a geometric point in terms of a coordinate system.
6. The branch of mathematics concerned with numerical calculations, such as addition, subtraction, multiplication, and division.
7. The branch of mathematics concerned with the foundations of mathematics.
8. The branch of mathematics concerned with the properties and interrelationships of sets.
9. The branch of mathematics concerned with the properties of trigonometric functions and their application to the determination of the angles and sides of triangles.
10. The branch of mathematics concerned with the properties, relationships, and measurement of points, lines, curves, and surfaces.
11. The branch of mathematics consisting of calculus and its higher developments.
12. The branch of mathematics in which arithmetical operations and relationships are generalized by using alphabetic symbols to represent unknown numbers or members of specified sets of numbers.
13. The branch of mathematics, developed independently by Newton and Leibniz.
14. The branch of modern geometry in which certain axioms of Euclidean geometry are restated.

15. The calculation, description, manipulation, and interpretation of the mathematical attributes of sets or populations too numerous or extensive for exhaustive measurements.
16. The geometry based on the definitions and axioms set out in “The Elements”.
17. The mathematical study of probability.
18. The study and use of the mathematical techniques to solve practical problems.
19. The study of abstract mathematical systems and structures, without necessarily having practical applications in mind.
20. The study of integers, their properties, and the relationship between integers.

Activity 42. Match the fields of mathematics with the key figures they are associated with.

1. algebra	a. Blaise Pascal
2. analytic geometry	b. Euclid
3. calculus	c. Évariste Galois
4. geometry	d. Georg Cantor
5. group theory	e. George Boole
6. mathematical logic	f. Gottfried Wilhelm Leibniz and Sir Isaac Newton
7. mathematical statistics	g. Henri Poincaré
8. number theory	h. Hipparchus
9. probability theory	i. Sir Ronald A. Fisher
10. set theory	j. Muhammad ibn Musa al-Khwarizmi
11. topology	k. Pierre de Fermat
12. trigonometry	l. René Descartes

Activity 43. In groups, choose one subject in the box to brainstorm. Exchange your ideas with other groups. Follow the instructions below.

1. Mathematics as a Basic Life Skill
2. Mathematics as a Compulsory School Subject
3. Mathematics as an Advanced University Course
4. Mathematics as a Fundamental Formal Science
5. Mathematics as an Active Research Area

The basics of brainstorming

Brainstorming is a group problem-solving method that involves the spontaneous contribution of creative ideas and solutions. This technique requires intensive, freewheeling discussion in which every member of the group is encouraged to think aloud and suggest as many ideas as possible based on their diverse knowledge.

It is a method of generating ideas and sharing knowledge to solve a particular problem, in which participants are encouraged to think without interruption. Brainstorming is a group activity where each participant shares their ideas as soon as they come to mind.

Brainstorming rules

1. Focus on quantity. You've likely heard the phrase “quality over quantity,” but when it comes to brainstorming, the exact opposite is true.
2. Withhold criticism. Negativity has no place in a brainstorming session.
3. Welcome unusual ideas.
4. Combine and improve ideas.

“Mathematics is the queen of sciences and number theory is the queen of mathematics.”

(Carl Friedrich Gauss)