"Mathematics is the queen of sciences and number theory is the queen of mathematics."

(Carl Friedrich Gauss)

Module 6. Mathematics and Related Fields

Unit 26. Physics and Physical Science

Activity 199. In pairs, discuss the questions.

1. Does a mathematician have to excel at physics?
2. Does a physicist have to excel at mathematics?
3. What is the difference between physics and physical science?
4. How are physics and mathematics related?

Activity 200. Match the words with the definitions.

1. matter
2. motion
3. energy

a. the capacity of a body or system to do work
b. the physical part of the universe consisting of solids, liquids, and gases
c. the process of continual change in the physical position of an object

Activity 201. Complete the tables.

Table 32. Base SI Units

amount of a substance / angle / electric current / length / light intensity / mass / solid angle / temperature / time		
Base Units	**Name**	**Symbol**
(1) ___________	meter	m

(2) ___________	kilogram	kg
(3) ___________	second	s
(4) ___________	Kelvin	K
(5) ___________	mole	mol
(6) ___________	ampere	A
(7) ___________	candela	cd
(8) ___________	radian	rad
(9) ___________	steradian	sr

Table 33. Conversion Between Metric and Imperial Systems

1,000 / 1,609 / 30 / 2.54 / 0.91 / 236.588 / 568.261 / 453.592 / 946.353 / 28.3495 / 29.5735 / 3.78541	
Metric System	**Imperial System**
Distance	
(10) _____ cm	1 in (inch) / 1”
(11) _____ cm	1 ft (foot) / 1’ = 12 in
(12) _____ m	1 yd (yard) = 3 ft
(13) _____ km	1 mi (mile) = 1,760 yd
Mass	
(14) _____ g	1 oz (ounce)
(15) _____ g	1 lb (pound) = 16 oz
(16) _____ kg	1 ton = 2204,62 lb
Volume	
(17) _____ ml	1 fl oz (fluid ounce)
(18) _____ ml	1 cup = 8 fl oz
(19) _____ ml	1 pt (pint) = 2.4019 cup
(20) _____ ml	1 qt (quart) = 2 pt
(21) _____ l	1 gal (gallon) = 4 qt

Activity 202. Read the article. Expand on the physical and abstract nature of mathematics as a science.

Physics is a science that deals with the structure of matter and the interactions between the fundamental constituents of the observable universe. In the broadest sense, physics is concerned with all aspects of nature on both the macroscopic and submicroscopic levels. Its scope of study encompasses not only the behaviour of objects under the action of given forces but also the nature and origin of gravitational, electromagnetic, and nuclear force fields. Its ultimate objective is the formulation of a few comprehensive principles that bring together and explain all such disparate phenomena.

Physics is the basic physical science. Until rather recent times physics and natural philosophy were used interchangeably for the science whose aim is the discovery and formulation of the fundamental laws of nature. As the modern sciences developed and became increasingly specialized, physics came to denote that part of physical science not included in astronomy, chemistry, geology, and engineering. Physics plays an important role in all the natural sciences, however, and all such fields have branches in which physical laws and measurements receive special emphasis, bearing such names as astrophysics, geophysics, biophysics, and even psychophysics.

Physics can, at base, be defined as the science of matter, motion, and energy. Its laws are typically expressed with economy and precision in the language of mathematics. Although mathematics is used throughout the physical sciences, it is often debated whether mathematics is itself a physical science. Those who include it as a physical science point out that physical laws can be expressed in mathematical terms and that the concept of number arises in counting physical objects. Those who say mathematics is not a physical science consider numbers as abstract concepts that are helpful in describing groups of objects but do not arise from the physical objects themselves.

The ultimate aim of physics is to find a unified set of laws governing matter, motion, and energy at small (microscopic) subatomic distances, at the human (macroscopic) scale of everyday life, and out to the largest distances (e.g., those on the extragalactic scale). This ambitious goal has been realized to a notable extent. Although a completely unified theory of physical phenomena has not yet been achieved (and possibly never will be), a remarkably small set of fundamental physical laws appears able to account for all known phenomena. The body of physics developed up to about the turn of the 20th century, known as classical physics, can largely account for the motions of macroscopic objects that move slowly with respect to the speed of light and for such phenomena as heat, sound, electricity, magnetism, and light. The modern developments of relativity and quantum mechanics modify these laws insofar as they apply to higher speeds, very massive objects, and to the tiny elementary constituents of matter, such as electrons, protons, and neutrons.

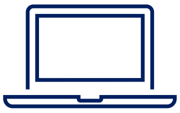
(from Encyclopaedia Britannica)

Table 34. Multiples of Units

Multiples	Prefix	Symbol	Origin
10^{24}	yotta-	Y	Greek “eight times” (24 = 8 × 3)
10^{21}	zetta-	Z	Greek “seven times” (21 = 7 × 3)
10^{18}	exa-	E	Greek “six times” (18 = 6 × 3)
10^{15}	peta-	P	Greek “five times” (15 = 5 × 3)
10^{12}	tera-	T	Greek “monster”
10^{9}	giga-	G	Greek “giant”
10^{6}	mega-	M	Greek “big”
10^{3}	kilo-	k	Greek “thousand”
10^{2}	hecto-	h	Greek “hundred”
10	deka-	da	Greek “ten”
10^{-1}	deci-	d	Latin “ten”
10^{-2}	centi-	c	Latin “hundred”
10^{-3}	milli-	m	Latin “thousand”
10^{-6}	micro-	μ	Greek “small”
10^{-9}	nano-	n	Greek “dwarf”
10^{-12}	pico-	p	Italian “small”
10^{-15}	femto-	f	Danish “15”
10^{-18}	atto-	a	Danish “18”
10^{-21}	zepto-	z	Greek “seven times”
10^{-24}	yocto-	y	Greek “eight times”

Activity 203. Finish the sentences from the text in Activity 202.

1. Physics is a science that deals with…
2. In the broadest sense, physics is concerned with…
3. Its scope of study encompasses…
4. Its ultimate objective is…
5. Physics plays an important role in…
6. Physics can, at base, be defined as…
7. Its laws are typically expressed…
8. The ultimate aim of physics is…



Activity 204. Do you believe that mathematics was discovered or created? Watch the video "Is Math Discovered or Invented?" to choose the best answer to the questions. Then watch the video again and make a note of all the proponents advocating for each point of view.

https://disk.yandex.ru/i/9_dzLe74Bp20pg

1. What did the Pythagoreans believe about numbers?
A. Numbers were invented by humans to measure things
B. Numbers were living entities and universal principles
C. Numbers only existed in mathematical equations
D. Numbers were useful but had no real meaning

2. What was Leopold Kronecker's view on mathematics?
A. All mathematical concepts were created by God
B. Mathematics exists independently in nature
C. Only natural numbers were divine; everything else was created by humans
D. Mathematics was discovered through scientific experiments

3. What does the phrase "the unreasonable effectiveness of mathematics" suggest?
A. Mathematics is too difficult for most people to understand
B. Mathematical theories often describe physical phenomena they weren't designed to explain
C. Mathematics should not be used in physics
D. Mathematical rules are unreasonable and need to be changed

4. Why is Godfrey Hardy's work mentioned?
A. To show that theoretical mathematics can later prove useful in real-world applications
B. To demonstrate that mathematics is always practical
C. To prove that number theory has no value
D. To explain why he won a Nobel Prize in mathematics

5. What is the main purpose of the video?
A. To prove that mathematics was definitely invented by humans
B. To teach readers how to solve mathematical problems
C. To present different perspectives on whether mathematics is discovered or invented
D. To explain why ancient mathematicians were wrong about numbers

Activity 205. In writing, comment on the citation.

Figure 10. Eugene Wigner

"The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning."

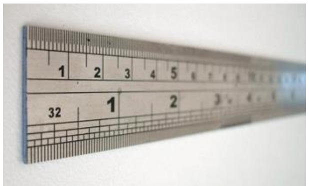
(by Eugene Wigner, from "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," 1960)

Unit 27. Computer Science

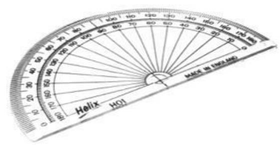
Activity 206. Match the words with the images.

1) a compass
   (a pair of compasses)
2) a protractor
3) a ruler
4) a set square (BrE)
   a triangle (AmE)

a)



b)



c)

d)

Activity 207. Match the words with the definitions.

1. abacus
2. calculator
3. computer

a. a device that is able to store a number, add it to another number, and mechanically produce the result, taking care of any carried digits
b. an electronic device for automatically performing either arithmetic operations on data or sequences of manipulations on sets of symbols (as required for algebra and set theory, for instance), all according to a precise set of predetermined instructions
c. any counting board with beads laid in parallel grooves or strung on parallel rods

Activity 208. In pairs, discuss the questions.

1. Is the computer an integral instrument of a mathematician? Why?
2. What tools and devices do mathematicians use? Why?
3. What computer software and smartphone applications do mathematicians use? Why?
4. How are computer science and mathematics related?

Activity 209. Read the article to expand on the interdisciplinary connection and mutual influence that exists between mathematics and computer science.

The roots of modern computer science lie in an interest in rapid computation. Simple mechanical calculators may date back to ancient times; however, it is the work of mathematicians Blaise Pascal (1623–1662) and Gottfried Leibniz (1646–1716) that gave rise to the first practical mechanical calculators. By the mid-19th century, Charles Babbage (1791–1871) had conceptualized and designed mechanical computers that included the essential features (programs, processor, memory, input/output) of the modern digital computer. His motivation was the need for rapid, accurate calculation of statistical tables made necessary by the manufacturing economy of the Industrial Revolution. By the end of the century, the volume of such data had increased to the point where mechanical calculators and tabulators had become the only practical way to keep up.

Mathematically, a computer can be seen as a way to rapidly and automatically execute procedures that have been proven to lead to reliable solutions to a problem. Once computers came on the scene, mathematical principles for verifying or proving algorithms would acquire new practical importance.

By the early 20th century, however, mathematicians were beginning to examine the problem of determining what propositions were provable, and in 1931 Kurt Gödel published a proof that any mathematical system necessarily allowed for the formation of propositions that could not be proven using the axioms of that system. An analogous question was determining what problems were computable. Working independently, two researchers formulated models that could be used to test for computability. Turing's model, in particular, provided a theoretical construct that could, using combinations of a few simple operations, calculate anything that was computable.

By the 1940s, electromechanical (relays) or electronic (tube) switching elements made it possible to build practical high-speed computers. Computer circuit designers could draw upon the advances in symbolic logic in the 19th century. Boolean logic, with its true/false values, would prove ideal for operating computers constructed from on/off switched elements.

The mathematical tools of the previous 150 years could now be used to design systems that could not only calculate but also manipulate symbols and achieve results in higher mathematics.

A variety of mathematical disciplines bear upon the design and use of modern computers. Simple or complex algebra using variables in formulas is at the heart of many programs ranging from financial software to flight simulators.

Geometry, particularly the analytical geometry based upon the coordinate system devised by René Descartes (1596–1650) is fundamental to computer graphics displays, where the screen is divided into X (vertical) and Y (horizontal) axes. Modern graphics systems have added 3D depiction and sophisticated algorithms to allow the rapid display of complex objects. Beyond graphics, the Cartesian insight that converted geometry into algebra makes a variety of geometrical problems accessible to computation, including the finding of optimum paths for circuit design. Design of computer and network architectures also involves the related field of topology. The fascinating field of fractal geometry has found use in computer graphics and data storage techniques.

Aspects of number theory, often considered the most abstract branch of mathematics, have found surprising relevance in computer applications. These include randomization (random number generation) and the factoring of large numbers, which is crucial for cryptography.

Mathematics as a discipline is thus essential to its younger sibling, computer science. In turn, however, computer science and technology have enriched the pursuit of mathematical truth in surprising ways. As early as 1956, a program called Logic Theorist, written by Herbert Simon (1916–2001) and Allen Newell (1927–1992) demonstrated how a program (that is, a collection of algorithms) could prove mathematical propositions given axioms and rules. While these early programs worked on a somewhat hit-or-miss basis, later theorem-solving programs produced solutions different from the standard ones known to mathematicians, and sometimes more elegant. Thus, the computer, which began as an aid to calculation, became an aid to symbol manipulation and to some extent an independent creative source.

(by Harry Henderson, from Encyclopedia of Computer Science and Technology)

Activity 210. Reorder the sentences to make a text on cryptography.

a. If letters of the alphabet and punctuation marks are replaced by numbers, then mathematics can be used to create effective codes.

b. In 1977 three mathematicians, Ron Rivest, Adi Shamir, and Leonard Adleman, developed a public-key cryptography method in which the method of encoding a message can be public to all without compromising the security of the message.
c. It is the primary encryption method used today by financial institutions to transmit sensitive information across the globe.
d. On the other hand, multiplying and raising large numbers to powers is easy for computers to do, and so the RSA method is also very easy to implement.
e. The practice of altering the form of a message by codes and ciphers to conceal its meaning to those who intercept it, but not to those who receive it, is called cryptography.
f. The RSA encryption method, as it is known today, is based on the mathematics of the modular arithmetic and relies on the fact that it is extraordinarily difficult to find the two factors that produce a given large product.
g. The RSA system is extremely secure.

Activity 211. What is natural language processing, speech recognition, and speech synthesis? Watch the video "The Turing Test: Can a Computer Pass For a Human?" to choose the best answer to the questions. Describe the purpose, procedure, and participants of the test.

https://disk.yandex.ru/i/vZNyknipXUWCUQ

1. What was Alan Turing's main approach to measuring artificial intelligence?
A. He focused on whether machines could develop consciousness
B. He tested whether computers could communicate like humans
C. He measured the memory capacity of different computers
D. He studied the neurons in artificial brains

2. Why did ELIZA successfully fool many people during conversations?
A. It had more memory than other programs at that time
B. It used complex mathematical equations to generate responses
C. It acted like a psychologist and reflected questions back to users
D. It could discuss any topic with detailed knowledge

3. What weakness of the Turing Test do ELIZA and PERI demonstrate?
A. Computers need too much memory to pass the test
B. People often think things are intelligent when they really aren't
C. Judges always know when they're talking to a machine
D. Programs can't have conversations about multiple topics

4. Why does human conversation remain difficult for computers?
A. Modern computers don't have enough processing power yet
B. Language involves complex understanding that goes beyond dictionary definitions
C. Programmers haven't created large enough databases of conversations
D. Computers can't remember previous conversations with users

5. What can be inferred about Turing's prediction for the year 2000?
A. He accurately estimated the memory computers would need
B. He underestimated how difficult human conversation would be for machines
C. He knew that computers would focus on fooling judges
D. He expected machines to develop consciousness by that time

Activity 212. Choose one technological innovation applicable to mathematical research and education. Present its functionality with multimedia slides.

Unit 28. Economics

Activity 213. Choose the best alternative.

1. A small car is more economic / economical to run.
2. Buy the large economy / economical pack.
3. Economic / Economical growth is slow.
4. He bought the economy / economical size and saved money.
5. He is studying economics / economy.
6. He thought I had been economic / economical with the truth.
7. Tourism is an important part of the economics / economy.

Activity 214. Complete the passage with the words in the box.

distribute / goods and services / market / produce / rate / resources / scarce / supply and demand

In the realm of mathematics, the principles of (1) __________ find a unique application. Much like in a (2) __________ where the availability of (3) __________ must meet consumer demand, mathematical problems involve a balance of resources and requirements. Equations and formulas act as the tools to (4) __________ solutions, with the (5) __________ at which these solutions are obtained reflecting the efficiency of mathematical processes. (6) __________, such as time and computational power, are often (7) __________, prompting mathematicians to (8) __________ their efforts wisely.

Activity 215. Read the passage. In pairs, discuss the questions in the box.

Economics is a social science that analyzes and describes the consequences of choices made concerning scarce productive resources. Economics is the study of how individuals and societies choose to employ those resources: what goods and services will be produced, how they will be produced, and how they will be distributed among the members of society. Economics is customarily divided into microeconomics and macroeconomics. Of major concern to macroeconomists are the rate of economic growth, the inflation rate, and the rate of unemployment. Specialized areas of economic investigation attempt to answer questions on a variety of economic activity; they include agricultural economics, economic development, economic history, environmental economics, industrial organization, international trade, labour economics, money supply and banking, public finance, urban economics, and welfare economics. Specialists in mathematical economics and econometrics provide tools used by all economists. The areas of investigation in economics overlap with many other disciplines, notably history, mathematics, political science, and sociology.

(from Encyclopaedia Britannica)

1. What is economics?
2. What branches does economics comprise?
3. What role does mathematics play in economics?

Activity 216. Read the article to match the headings (a–c) with the sections (1–3).

(a) Applications of mathematics in economics

(b) Branches of mathematics integral to economics

(c) Role of mathematics in economics

(1) ________________________________

Although mathematics is indispensable to all types of economics, it's most common in mathematical economics, where it's a core component. In mathematical economics, economists apply mathematical principles to economic theory. An economist may use mathematics alongside other methods and tools and techniques, such as data harvesting and computer algorithms.

Mathematics in economics allows an economist to offer more precision with their projections and analysis. This may allow them to extract increased guidance from the results of their analysis. Using hard data and mathematical calculations can also reduce the potential for bias in economic projections.

The importance of mathematics in economics has increased with the growing presence of computing in the field. Computer technology allows economists to process large amounts of data or more complex mathematical equations more easily. This expands the capability of mathematics in economics as computers can make complex calculations easier to complete.

(2) ______________________________

Algebra is a basic math field, and it serves as a foundation for many other forms of mathematical calculation. Algebra allows an individual to solve equations with one or more variables, finding the result for a variable under defined conditions. For an economist, algebra is a useful mathematical skill for calculation and projection. Working with variables allows an economist to perform a task such as setting a target growth rate and solving for the required related variables to reach that rate using an established equation.

Calculus is a mathematical field dealing with rate-of-change calculations. Calculus can be a powerful tool for an economist when assessing economic performance and making projections. Using calculus to generate curves based on economic information allows one to identify trends and make more informed decisions. As an economist, one may apply this to projects such as market assessment, supply and demand analysis and economic forecasting.

The mathematical field of probability measures the likelihood of a specific occurrence or outcome. Probabilistic assessment allows a mathematician to identify whether a potential outcome is likely to occur and to compare the relative likelihood of two or more potential outcomes. As an economist, understanding probability can be useful when making estimations of likely outcomes to economic decisions. By combining the probability of different potential outcomes multiplied by the estimated cost or benefit of each outcome, one can perform an assessment of the expected cost or benefit of an economic plan.

Statistics is a mathematical study that focuses on the collection, sorting and analysis of sets of data. It allows a mathematician to assess a population represented within the data. That is a critical skill for tasks such as modelling and projecting for behaviours or responses within a community. Statistics is a valuable mathematical skill for an economist because it allows one to work with large amounts of data. Applying statistical calculations to datasets may allow an economist to identify key trends or information on grouping within a community. One may then use this information to guide oneself or others in decision-making positions when making suggestions for plans or policies.

(3) ______________________________

Analysis is a key responsibility for an economics professional. Economic work often includes assessing information about economic performance, markets and other key economic data and extrapolating relevant information from the data. This allows individuals making economic decisions to do so while using information from various sources. Math plays a large part in many forms of data analysis. This can include both the simple mathematics to perform tasks such as finding averages to advanced mathematics in the form of differential equations. Strong math skills in a diverse range of math capabilities can help an economist complete their analytical work more effectively.

An economic model provides a visualization of key economic data. Using a model can make it easier for individuals to conceptualize or understand the state of an economic market. A model may also provide a new form for data that offers insights that the raw dataset does not. An economist is likely to use their math skills throughout the process of creating an economic model. Accurate math provides reliable data one can use in constructing a model, which can increase the value of the model provides upon completion.

Economic projections provide predictions of future economic behaviour and patterns. Accurate projections are a valuable tool for economists, as it allows them to make decisions for future planning based on the state and behaviour of the market in the future. Math is an integral part of creating economic projections. It allows an economist to perform calculations on economic data, often using the principles of calculus to assess potential changes in the data over time. Developing mathematical skills as an economist can help improve the accuracy of calculations both by ensuring one completes them correctly and expanding the number of calculations and math principles one understands and may apply to one's work.

(from indeed.com)

Activity 217. Outline mathematics in economics by developing the points.

1. mathematical economics
2. computers
3. algebra
4. calculus
5. probability
6. statistics
7. analyzing
8. modelling
9. projecting

Activity 218. Reorder the sentences to make a text on econometrics.

a. Econometricians estimate production functions and cost functions for firms, supply-and-demand functions for industries, income distribution in an economy, macroeconomic models and models of the monetary sector for policy makers, and business cycles and growth for forecasting.
b. Econometrics creates equations to describe phenomena such as the relationship between changes in price and demand.
c. Econometrics is the statistical and mathematical analysis of economic relationships, often serving as a basis for economic forecasting.
d. Information derived from these models helps both private businesses and governments make decisions and set monetary and fiscal policy.
e. It is used mainly, however, by economists to study relationships between economic variables.
f. Such information is sometimes used by governments to set economic policy and by private business to aid decisions on prices, inventory, and production.

Activity 219. Draw a mind map of the text in Activity 216. You can use the following resources:

https://app.diagrams.net/

https://freemind.sourceforge.net/wiki/index.php/Main_Page

https://simplemind.eu/

https://coggle.it/

https://www.mindmup.com/

https://miro.com/ru/

Unit 29. Mathematics in Non-Fiction

Activity 220. In pairs, discuss the questions.

1. Why did you decide to specialize in mathematics?
2. Did you dream of becoming a mathematician?
3. Did you like your teacher(s) of mathematics? Why?
4. How has your attitude to mathematics changed over the years?

Activity 221. Match the influential mathematical books with their authors. Characterize the contents and impact of one text.

1. A Course of Pure Mathematics
2. Arithmetica
3. Ars Magna
4. Elements of Mathematics
5. La Géométrie
6. Liber Abaci
7. Principia Mathematica
8. Statistical Methods for Research Workers
9. Synagoge / Collection
10. The Almagest
11. The Compendious Book on Calculation by Completion and Balancing / Al-Jabr
12. The Doctrine of Chances
13. The Elements
14. The Ground of Arts
15. The Laws of Thought
16. Theory of Games and Economic Behavior

a. Abraham de Moivre
b. Alfred North Whitehead and Bertrand Russell
c. Claudius Ptolemy
d. Diophantus of Alexandria
e. Euclid
f. Fibonacci
g. G. H. Hardy
h. George Boole
i. Gerolamo Cardano
j. John von Neumann and Oskar Morgenstern
k. Muhammad ibn Musa al-Khwarizmi
l. Nicolas Bourbaki
m. Pappus of Alexandria
n. René Descartes
o. Robert Recorde
p. Sir Ronald Aylmer Fisher

Activity 222. What is a polymath? Watch the video "The Greatest Mathematician That Never Lived" to choose the best answer to the questions. Write a summary of Nicolas Bourbaki's identity and mathematical legacy.

https://disk.yandex.ru/i/R3HeCOX51AvpoA

1. Why was Nicolas Bourbaki's application to the American Mathematical Society rejected?
A. His published work was not considered important enough
B. He was not a real person
C. He refused to meet with the committee members
D. His textbooks contained too many errors

2. What was the main problem with mathematics before the group formed?
A. There were too many mathematicians working in the field
B. Mathematical textbooks were too expensive for students
C. Different branches of mathematics lacked a common framework
D. Universities refused to teach advanced mathematical concepts

3. What does the is suggested about the group's approach to mathematics?
A. They believed mathematics should be based on intuition rather than logic
B. They thought that strict logical systems would limit mathematical creativity
C. They challenged the common view that mathematics was primarily intuitive
D. They wanted to separate mathematics into more specialized branches

4. What is a bijective function?
A. A function where multiple inputs can produce the same output
B. A function where each element has a one-to-one correspondence
C. A function where outputs cannot be mapped back to inputs
D. A function that only works with numerical values

5. How did the group maintain the illusion that Bourbaki was real?
A. They hired an actor to appear at mathematical conferences
B. They created elaborate stories and sent communications in his name
C. They published false biographical information in academic journals
D. They claimed he had won several international mathematics awards

Activity 223. Read the excerpt from the 1940 essay "A Mathematician's Apology" by British mathematician G. H. Hardy. Compare your mathematical journey to that of G. H. Hardy. Are there ideas in the text that resonate with you? Identify any similarities and/or differences between you and the author. In pairs, exchange your thoughts.

Figure 11. G. H. Hardy

I cannot remember ever having wanted to be anything but a mathematician. I suppose that it was always clear that my specific abilities lay that way, and it never occurred to me to question the verdict of my elders. I do not remember having felt, as a boy, any passion for mathematics, and such notions as I may have had of the career of a mathematician were far from noble. I thought of mathematics in terms of examinations and scholarships: I wanted to beat other boys, and this seemed to be the way in which I could do so most decisively.

I found at once, when I came to Cambridge, that a Fellowship implied "original work", but it was a long time before I formed any definite idea of research. I had of course found at school, as every future mathematician does, that I could often do things much better than my teachers; and even at Cambridge, I found, though naturally much less frequently, that I could sometimes do things better than the College lecturers. But I was really quite ignorant, even when I took the Tripos of the subjects on which I have spent the rest of my life; and I still thought of mathematics as essentially a "competitive" subject. My eyes were first opened by Professor Love, who taught me for a few terms and gave me my first serious conception of analysis. But the great debt which I owe to him was his advice to read Jordan's famous "Cours d'Analyse"; and I shall never forget the astonishment with which I read that remarkable work, the first inspiration for so many mathematicians of my generation, and learnt for the first time as I read it what mathematics really meant. From that time onwards, I was in my way a real mathematician, with sound mathematical ambitions and a genuine passion for mathematics.

I wrote a great deal during the next ten years, but very little of any importance; there are not more than four or five papers which I can still remember with some satisfaction. The real crisis of my career came ten or twelve years later, in 1911, when I began my long collaboration with Littlewood, and in 1913, when I discovered Ramanujan. All my best work since then has been bound up with theirs, and it is obvious that my association with them was the decisive event of my life. I still say to myself when I am depressed and find myself forced to listen to pompous and tiresome people, "Well, I have done one the thing you could never have done, and that is to have collaborated with both Littlewood and Ramanujan on something like equal terms." It is to them that I owe an unusually late maturity: I was at my best a little past forty, when I was a professor at Oxford. Since then I have suffered from that steady deterioration which is the common fate of elderly men and particularly of elderly mathematicians. A mathematician may still be competent enough at sixty, but it is useless to expect him to have original ideas.

It is plain now that my life, for what it is worth, is finished, and that nothing I can do can perceptibly increase or diminish its value. It is very difficult to be dispassionate, but I count it a "success"; I have had more reward and not less than was due to a man of my particular grade of ability. I have held a series of comfortable and "dignified" positions. I have had very little trouble with the duller routine of universities. I hate "teaching", and have had to do very little, such teaching as I have done been almost entirely supervision of research; I love lecturing and have lectured a great deal to extremely able classes; and I have always had plenty of leisure for the research which has been the one great permanent happiness of my life. I have found it easy to work with others and have collaborated on a large scale with two exceptional mathematicians; and this has enabled me to add to mathematics a good deal more than I could reasonably have expected. I have had my disappointments, like any other mathematician, but none of them has been too serious or has made me particularly unhappy. If I had been offered a life neither better nor worse when I was twenty, I would have accepted without hesitation.

It seems absurd to suppose that I could have "done better". I have no linguistic or artistic ability, and very little interest in experimental science. I might have been a tolerable philosopher, but not one of a very original kind. I think that I might have made a good lawyer; but journalism is the only profession, outside academic life, in which I should have felt really confident of my changes. There is no doubt that I was right to be a mathematician, if the criterion is to be what is commonly called success.

My choice was right, then, if what I wanted was a reasonably comfortable and happy life. But solicitors and stockbrokers and bookmakers often lead comfortable and happy lives, and it is very difficult to see how the world is richer for their existence. Is there any sense in which I can claim that my life has been less futile than theirs? It seems to me again that there is only one possible answer: yes, perhaps, but, if so, for one reason only:

I have never done anything "useful". No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world. I have helped to train other mathematicians, but mathematicians of the same kind as myself, and their work has been, so far at any rate as I have helped them to it, as useless as my own. Judged by all practical standards, the value of my mathematical life is nil; and outside mathematics it is trivial anyhow. I have just one chance of escaping a verdict of complete triviality, that I may be judged to have created something worth creating. And that I have created is undeniable: the question is about its value.

The case for my life, then, or for that of any one else who has been a mathematician in the same sense which I have been one, is this: that I have added something to knowledge, and helped others to add more; and that these somethings have a value which differs in degree only, and not in kind, from that of the creations of the great mathematicians, or of any of the other artists, great or small, who have left some kind of memorial behind them.

(from "A Mathematician's Apology," by G. H. Hardy, 1940)

Activity 224. In groups, write an essay "A Message in a Bottle to Fellow Mathematicians" collating your thoughts and experiences, advice and lessons that you wish to pass down to future generations of mathematicians. Exchange your essays with other groups.

Activity 225. "Mathematicians are born, not made" (Henri Poincaré). Make a list of arguments either supporting or opposing the statement. Debate in groups.

Activity 226. In pairs or groups, consult Appendix III and choose two or three mathematicians. Dramatize a talk-show interview between the chosen mathematicians, discussing "your" lives and work.

Unit 30. Mathematics in Fiction

Activity 227. In pairs, discuss the questions.

1. Are you a creative person?
2. Does a mathematician need to be creative? Why?
3. Is mathematics an art form? Why?
4. Do you think the fields of language arts and mathematics are related? If so, how?

Activity 228. In groups, comment on the quotes.

1. “A mathematical equation stands forever.” (Albert Einstein)
2. “Algebra is the intellectual instrument which has been created for rendering clear the quantitative aspect of the world.” (Alfred North Whitehead)
3. “All the effects of nature are only mathematical results of a small number of immutable laws.” (Pierre-Simon Laplace)
4. “Besides language and music, mathematics is one of the primary manifestations of the free creative power of the human mind.” (Hermann Weyl)
5. “Geometry will draw the soul toward truth and create the spirit of philosophy.” (Plato)
6. “In mathematics I can report no deficiency, except it be that men do not sufficiently understand the excellent use of the pure mathematics.” (Francis Bacon)
7. “In mathematics the art of proposing a question must be held of higher value than solving it.” (Georg Cantor)
8. “In mathematics you don't understand things. You just get used to them.” (John von Neumann)
9. “It is impossible to be a mathematician without being a poet in soul.” (Sofya Kovalevskaya)
10. “Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them.” (Joseph Fourier)
11. “Mathematics is as much an aspect of culture as it is a collection of algorithms.” (Carl Benjamin Boyer)
12. “Mathematics is no more computation than typing is literature.” (John Allen Paulos)
13. “Mathematics is not only real, but it is the only reality.” (Martin Gardner)
14. “Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.” (David Hilbert)
15. “Mathematics rightly viewed possesses not only truth but supreme beauty.” (Bertrand Russell)
16. “Mighty is geometry; joined with art, resistless.” (Euripides)
17. “Numbers constitute the only universal language.” (Nathanael West)

18. “Probability theory is nothing but common sense reduced to calculation.” (Pierre-Simon Laplace)
19. “Pure mathematics is, in its way, the poetry of logical ideas.” (Albert Einstein)
20. “The highest form of pure thought is in mathematics.” (Plato)
21. “The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics.” (G. H. Hardy)
22. “The study of mathematics, like the Nile, begins in minuteness but ends in magnificence.” (Charles Caleb Colton)
23. “When you can measure what you are talking about and express it in numbers, you know something about it.” (Lord Kelvin)
24. “Where there is matter, there is geometry.” (Johannes Kepler)
25. “Wherever there is a number, there is beauty.” (Proclus)

Activity 229. Read the extract from the 2003 novel “The Housekeeper and the Professor” by Japanese writer Yoko Ogawa. Explain the concept of “amicable numbers” based on the text.

"Do you send a lot of articles to magazines?" I asked.

"I wouldn't call them 'articles.' They're just puzzles for amateur mathematicians. Sometimes there's even a prize. Wealthy men who love mathematics put up the money." He looked down, checking his suit in various places, and his gaze fell on a note clipped to his left pocket. "Oh, I see. I sent a proof to the Journal of Mathematics today."

It had been much more than eighty minutes since I'd made my trip to the post office.

"Oh, dear!" I said. "If it's a contest, I should have sent it express mail. If it doesn't get there first, I suppose you don't get the prize."

"No, there was no need to send it express. Of course, it's important to arrive at the correct answer before anyone else, but it's just as important that the proof is elegant."

"I had no idea a proof could be beautiful... or ugly."

"Of course, it can," he said. Getting up from the table, he came over to the sink where I was washing the dishes and peered at me as he continued. "The truly correct proof is one that strikes a harmonious balance between strength and flexibility. There are plenty of proofs that are technically correct but are messy and inelegant or counterintuitive. But it's not something you can put into words — explaining why a formula is beautiful is like trying to explain why the stars are beautiful."

I stopped washing and nodded, not wanting to interrupt the Professor's first real attempt at conversation.

"Your birthday is February twentieth. Two twenty. Can I show you something? This was a prize I won for my thesis on transcendent number theory when I was at college." He took off his wristwatch and held it up for me to see. It was a stylish foreign brand, quite out of keeping with the Professor's rumpled appearance.

"It's a wonderful prize," I said.

"But can you see the number engraved here?" The inscription on the back of the case read President's Prize No. 284.

"Does that mean that it was the two hundred and eighty-fourth prize awarded?"

"I suppose so, but the interesting part is the number 284 itself. Take a break from the dishes for a moment and think about these two numbers: 220 and 284. Do they mean anything to you?"

Pulling me by my apron strings, he sat me down at the table and produced a pencil stub from his pocket. On the back of an advertising insert, he wrote the two numbers, separated strangely on the card.

220

284

"Well, what do you make of them?"

I wiped my hands on my apron, feeling awkward, as the Professor looked at me expectantly. I wanted to respond but had no idea what sort of answer would please a mathematician. To me, they were just numbers.

"Well...," I stammered. "I suppose you could say they're both three-digit numbers. And that they're fairly similar in size — for example, if I were in the meat section at the supermarket, there'd be very little difference between a package of sausage that weighed 220 grams and one that weighed 284 grams. They're so close that I would just buy the one that was fresher. They seem pretty much the same — they're both in the two hundreds, and they're both even —"

"Good!" he almost shouted, shaking the leather strap of his watch. I didn't know what to say. "It's important to use your intuition. You swoop down on the numbers, like a kingfisher catching the glint of sunlight on the fish's fin." He pulled up a chair, as if wanting to be closer to the numbers. The musty paper smell from the study clung to the Professor.

"You know what a factor is, don't you?"

"I think so. I'm sure I learned about them at some point..."

"For 220 is divisible by 1 and by 220 itself, with nothing leftover. So, 1 and 220 are factors of 220. Natural numbers always have 1 and the number itself as factors. But what else can you divide it by?"

"By 2, and 10..."

"Exactly! So, let's try writing out the factors of 220 and 284, excluding the numbers themselves. Like this."

220 : 1 2 4 5 10 11 20 22 44 55 110

142 71 4 2 1 : 284

The Professor's figures, rounded and slanting slightly to one side, were surrounded by black smears where the pencil had smudged.

"Did you figure out all the factors in your head?" I asked.

"I don't have to calculate them — they just come to me from the same kind of intuition you used. So then, let's move on to the next step," he said, adding symbols to the lists of factors.

220 : 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 =

= 142 + 71 + 4 + 2 + 1 : 284

"Add them up," he said. "Take your time. There's no hurry."

He handed me the pencil, and I did the calculation in the space that was left on the advertisement. His tone was kind and full of expectation, and it didn't seem as though he were testing me. On the contrary, he made me feel as though I were on an important mission, that I was the only one who could lead us out of this puzzle and find the correct answer.

I checked my calculations three times to be sure I hadn't made a mistake. At some point, while we'd been talking, the sun had set, and night was falling. From time to time, I heard water dripping from the dishes I had left in the sink. The Professor stood close by, watching me.

"There," I said. "I'm done."

220 : 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284

220 = 142 + 71 + 4 + 2 + 1 : 284

"That's right! The sum of the factors of 220 is 284, and the sum of the factors of 284 is 220. They're called 'amicable numbers,' and they're extremely rare. Fermat and Descartes were only able to find one pair each. They're linked to each other by some divine scheme, and how incredible that your birthday and this number on my watch should be just such a pair."

We sat staring at the advertisement for a long time. With my finger I traced the trail of numbers from the ones the Professor had written to the ones I'd added, and they all seemed to flow together, as if we'd been connecting up the constellations in the night sky.

(from "The Housekeeper and the Professor,"

written by Yoko Ogawa, translated by Stephen Snyder,

2003)

Activity 230. Look at the list of mathematical concepts encountered in the novel "The Housekeeper and the Professor". Choose one and expand on it.

1. abundant number
2. amicable number
3. Artin's conjecture
4. deficient number
5. Euler's formula
6. factorial
7. Fermat's Last Theorem
8. imaginary number
9. Mersenne prime
10. Napier's constant
11. perfect number
12. prime number
13. root
14. Ruth-Aaron pair
15. triangular number
16. twin prime

Activity 231. Consult Appendix IV. Choose one film to watch a trailer of. In pairs or groups, role-play a scene from the trailer.

Activity 232. Choose one film from Appendix IV to watch. Write a review of the film. Include a summary of the plot, your impression of the film, and critical evaluation.