Activity 111. Read the article. Review your answers to the quiz in Activity 110.

The study of geometry is an ancient one. Records show that Egyptian and Babylonian scholars of around 1900 B.C.E. had developed sound principles of measurement and spatial reasoning in their architecture and in their surveying of land. Both cultures were aware of Pythagoras's theorem and had developed tables of Pythagorean triples. (The Egyptians used knotted ropes to construct "3-4-5 triangles" to create right angles.) Ancient Indian texts on altar construction and temple building demonstrate sophisticated geometry knowledge, and the famous volume "The Nine Chapters on the Mathematical Art" from ancient China also includes work on the Pythagorean theorem.

In ancient Greece, mathematical scholars came to realize that many properties of shapes and figures could be deduced logically from other properties. In his epic work "The Elements" the Greek geometer Euclid (ca. 300–260 B.C.E.) collated a large volume of knowledge on the subject and showed that each and every result could be logically deduced from a very small set of basic assumptions (self-evident truths) about how geometry should work. Euclid's work was rigorous and systematic, and the notion of a logical proof was born. Euclid's postulates and the process of logical reasoning became the model of all further geometric investigation for the two millennia that followed. His method of compiling and organizing all mathematical knowledge known at his time was a significant intellectual achievement. Euclid's rigorous approach was, and still is, modelled in other branches of mathematics. Scholars in set theory, the foundations of mathematics, and calculus, for instance, all seek to follow the same process of formal reasoning as the correct approach to achieve proper understanding of these topics.

The next greatest breakthrough in the advancement of geometry occurred in the 17^{th} century with the discovery of Cartesian coordinates as a means to represent points as pairs of real numbers and lines and curves as algebraic equations. This approach, described by the French mathematician and philosopher René Descartes (1596–1650) in his famous 1637 work "Geometry", united the then-disparate fields of algebra and geometry. Unfortunately, Descartes's interests lay only in advancing methods of geometric construction, not in developing a full algebraic model of geometry. This latter task was pursued by the French mathematician Pierre de Fermat (1601–65), who had also outlined the principles of coordinate geometry in an unpublished manuscript that he had circulated among mathematicians before the release of "Geometry". Fermat later published the work in 1679 under the title "On the Plane and Solid Locus". The application of algebra to the discipline provided scholars a powerful new tool for solving geometric problems, and also provided them with a large number of different types of curves for study.

Fermat's work in geometry inspired work on the theory of differential calculus and, later, led to the study of "differential geometry" (the application of calculus to the study of shapes and surfaces). This was developed by the German mathematician and physicist Carl Friedrich Gauss (1777–1855).

Neither Descartes nor Fermat permitted negative values for distances. Consequently, neither scholar worked with a full set of coordinate axes as we use them today. The notions of negative distance and negative area were first put forward by Sir Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716), the coinventors of calculus.

The 19th century saw other major advances in geometry. It had long been noted that Euclid's fifth postulate, the so-called parallel postulate, is not necessary for a great deal of geometry. Many Arab scholars of the first millennium attempted, without success, to show that the fifth postulate could be logically deduced from the remaining four (thereby rendering it unnecessary), as did European scholars of the Renaissance. In 1795 the Scottish mathematician John Playfair showed that the fifth postulate is equivalent to the statement that, through any point, one can draw one, and only one, line through that point parallel to a given line. (This is today called Playfair's axiom.) Although not eliminating the need for the fifth postulate, Playfair showed that it could be understood in a more tractable form.

In 1829 the Russian mathematician Nikolai Ivanovich Lobachevsky (1792–1856) took a bold step and considered a geometric world in which the fifth postulate is false. He assumed that through a given point more than one line could be drawn parallel to a given line. In doing this, Lobachevsky discovered a new, consistent mathematical system free from contradiction, one as logically valid as the geometry of Euclid. (This geometry is today called hyperbolic geometry.) The philosophical impact of Lobachevsky's work was enormous: he had shown that mathematics need not be based on a single set of physical truths, and that other equally valid mathematical systems do exist based on alternative, carefully chosen axioms. Lobachevsky had also shown that Euclid's fifth postulate cannot be established as a consequence of the remaining four axioms: he had presented a valid example of a system in which the first four of Euclid's postulates hold, but the fifth does not.

Surprisingly some of Lobachevsky's ideas were anticipated well before the 19th century. The great Persian mathematician and poet Omar Khayyam (ca. 1048–1122) established a number of results that we recognize today as non-Euclidean. These results were later translated into Latin, and extended upon, by the Italian priest Girolamo Saccheri (1667–1733). Unfortunately, neither scholar discovered the validity of non-Euclidean geometry, as each was focused instead on trying to establish Euclid's fifth postulate as a consequence of the remaining four.

The German mathematician Bernhard Riemann (1826–66) discovered an alternative form of non-Euclidean geometry in which Euclid's fifth postulate fails in a different way. In a system of spherical geometry, it is never possible to draw a line through a given point parallel to a given line.

Riemann's contributions to the advancement of geometry were significant. In his famous 1854 lecture "On the Hypotheses that Lie at the Foundation of Geometry", Riemann put forward the view that geometry can be the study of any kind of space of any number of dimensions, and later developed the mathematics needed to properly describe the shape of space. Albert Einstein (1879–1955) later used this work to develop his theory of relativity.

(by James Tanton, from Encyclopedia of Mathematics)