Activity 130. Read the article. In pairs, discuss the questions in the box.

Loosely speaking, a set is any collection of objects or numbers specified in a well-defined manner. Each item in the set is called an element, or a member, of the set. For example, “dog” is an element of the set of mammals. If an entity “a” is an element of a set S, we write $a \in S$. If “a” does not belong to S, we write $a \notin S$.

Sets are typically specified either by listing the elements of the set between a set of braces “{ }”, or listing a few elements of the set to indicate a pattern. For example {a, e, i, o, u} is the set consisting of the five vowels of the alphabet, and {3, 6, 9, 12, ...} is the set of all multiples of 3. It may also be possible to define a set as consisting of elements from some universal collection that satisfy a certain property. For example, $\{x \in R \mid x > 5\}$ denotes the set of all real numbers that are greater than 5. (Some mathematicians prefer to use a colon “:” instead of a vertical bar in this notation.)

The order in which the elements of a set are listed is immaterial. For example, {A, 6, *} and {*, 6, A} are the same set. Also, elements of a set are listed without repetition. For instance, the set {a, a, a, a, a} is the set with a single element “a”. The empty set (the null set, the void set) is the set that contains no elements.

Two sets are deemed equal if they possess precisely the same elements. For example, the sets {2, 4, 6, 8, ...} and {n | n is a counting number divisible by 2} are equal sets. A set A is said to be a subset of a set B if every element of A is also a member of B. We write $A \subset B$ if we are certain that the two sets are not equal, and $A \supseteq B$ if equality of the sets is possible. For example, the set of all multiples of 4 is a subset of the set of all multiples of 2.

Although the intuitive notion of a set as a collection of objects is as ancient as the human race, the idea of a set as a formal mathematical concept was not proposed until the 19th century. In his development of Boolean algebra, The British mathematician George Boole (1815–64) introduced the notion of set as a fundamental tool for the study of formal logic. The German mathematician Georg Cantor (1845–1918), in his attempts to understand the foundation of all of mathematics, came to regard sets as even more basic and fundamental than the notion of number. Cantor properly formalized a theory of set manipulations and introduced the striking notion of cardinality (the cardinality of a set is a measure of the number of elements of the set). His work led him to profound insights into the nature of finite and infinite sets, leading him to extend the concept of number to include more than one type of infinity.

Intuitively, a set is said to be finite if one can recite all the elements of the set in a bounded amount of time. For instance, the set {knife, fork, spoon} is finite, for it takes only a second or two to recite the elements of this set. On the other hand, the set of natural numbers {1, 2, 3, ...} is not finite, for one can never recite each and every element of this set.

Despite our intuitive understanding of the concept, it is difficult to give a precise and direct mathematical definition of a finite set. The easiest approach is to simply define a finite set to be one that is not infinite, since the notion of an infinite set can be made clear. Alternatively, since there is a well-defined procedure for mechanically writing down the string of natural numbers 1, 2, 3, ..., one can define a finite set to be any set S whose elements can be put in one-to-one correspondence with a bounded initial segment of the string of natural numbers. For instance, matching "knife" with 1, "fork" with 2, and "spoon" with 3, the set {knife, fork, spoon} is finite because its elements can be matched precisely with the string of natural numbers {1, 2, 3}.

In 1902 the British mathematician and philosopher Bertrand Arthur William Russell (1872–1970) stunned the mathematical community with his construction of a simple paradox, today called Russell's paradox, that shows that our naive understanding of the notion of set is fundamentally flawed. Although Cantor believed that set theory is the foundation on which all of mathematics is built, it became clear to mathematicians that the concept of a set and what it means to be an "element of" must remain as undefined terms. In the decades that followed, mathematicians such as Ernst Friedrich Ferdinand Zermelo (1871–1953) attempted to develop an axiomatic theory of sets (based on undefined terms) that successfully avoids Russell's paradox. To this day, not all mathematicians agree that this goal has yet been achieved.

(from Encyclopaedia Britannica)

1. What is a set?
2. How are sets typically specified?
3. What is the significance of the order in which elements are listed in a set?
4. What types of sets are there?
5. Who introduced the notion of sets?
6. How did George Boole use sets in the development of Boolean algebra?
7. What did Georg Cantor contribute to the understanding of sets and the concept of infinity?
8. How is the finiteness of a set intuitively described, and why is it challenging to provide a precise mathematical definition?