

Activity 137. Read the article. Determine whether the statements in the box below are true or false by quoting from the text.

In mathematics, the systematic study of reasoning is called formal logic or symbolic logic. It analyzes the structure of arguments, as well as the methods and validity of mathematical deduction and proof.

The principles of logic can be attributed to Aristotle (384–322 B.C.E.), who wrote the first systematic treatise on the subject. He sought to identify modes of inference that are valid by virtue of their structure, not their content. For example, “Green and blue are colours; therefore green is a colour” and “Cows and pigs are reptiles; therefore cows are reptiles” have the same structure (“A and B, therefore A”), and any argument made via this structure is logically valid. (In particular, the second example is logically sound.) This mode of thought allowed Euclid (ca. 300–260 B.C.E.) to formalize geometry, using deductive proofs to infer geometric truths from a small collection of axioms (self-evident truths).

No significant advance was made in the study of logic for the millennium that followed. This period was mostly a time of consolidation and transmission of the material from antiquity. The Renaissance, however, brought renewed interest in the topic. Mathematical scholars of the time, Pierre Hérigone and Johann Rahn in particular, developed means for representing logical arguments with abbreviations and symbols, rather than words and sentences. Gottfried Wilhelm Leibniz (1646–1716) came to regard logic as “universal mathematics.” He advocated the development of a “universal language” or a “universal calculus” to quantify the entire process of mathematical reasoning. He hoped to devise new mechanical symbolism that would reduce errors in thinking to the equivalent of arithmetical errors. (He later abandoned work on this project, assessing it too daunting a task for a single man.)

In the mid-1800s George Boole succeeded in creating a purely symbolic approach to propositional logic, that part which deals with inferences involving simple declarative sentences (statements) joined by the connectives: not, and, or, if ... then ..., iff (if and only if). (These are called the negation, conjunction, disjunction, conditional, and the biconditional, respectively.) He successfully applied it to mathematics, thereby making a significant step to achieving Leibniz’s goal.

In 1879 the German mathematician and philosopher Gottlob Frege constructed a symbolic system for predicate logic. This generalizes propositional logic by including quantifiers: statements using words such as “some”, “all”, and “no”. (For example, “All men are mortal” as opposed to “This man is mortal.”) At the turn of the century David Hilbert sought to devise a complete, consistent formulation of all of mathematics using a small collection of symbols with well-defined meanings. The English mathematician and philosopher Bertrand Russell, in collaboration with his colleague Alfred North Whitehead, took up Hilbert’s challenge. In 1925 they published a monumental work. Beginning with an impressively minimal set of premises (“self-evident” logical principles), they attempted to

establish the logical foundations of all of mathematics. This was an impressive accomplishment. (After hundreds of pages of symbolic manipulations, they established the validity of “ $1 + 1 = 2$,” for example.) Although they did not completely reach their goal, Russell and Whitehead’s work has been important for the development of logic and mathematics.

Six years after the publication of their efforts, however, Kurt Gödel stunned the mathematical community by proving Hilbert’s (and Leibniz’s) goal to be futile. He demonstrated once and for all that any formal system of logic sufficiently sophisticated to incorporate basic principles of arithmetic cannot attain all the statements it hopes to prove. His results are today called Gödel’s incompleteness theorems. The vision to reduce all truths of reason to incontestable arithmetic was thereby shattered.

(from Encyclopaedia Britannica)

1. Aristotle wrote the first treatise on logic, focusing on the content rather than the structure of arguments.
2. Bertrand Russell and Alfred North Whitehead's work in 1925 attempted to establish the logical foundations of mathematics.
3. David Hilbert aimed to formulate all of mathematics consistently, inspiring Russell and Whitehead's monumental 1925 work.
4. Formal logic, also known as symbolic logic, systematically studies reasoning in mathematics, analyzing argument structures and the validity of deductions and proofs.
5. George Boole's symbolic approach to propositional logic had no significant impact on mathematics.
6. In 1879, Gottlob Frege developed a symbolic system for predicate logic, extending propositional logic with quantifiers like "some" and "all."
7. Kurt Gödel's incompleteness theorems supported Hilbert's goal.
8. Leibniz regarded logic as "universal mathematics" and advocated for the development of a "universal language" in mathematical reasoning.
9. The Renaissance was a period of decline for logical studies.
10. The vision to reduce all truths of reason to incontestable arithmetic remains intact despite Gödel's results.