Activity 167. Read the article. Review your answers to the quiz in Activity 166.

The Babylonians of 2000 B.C.E. lived in Mesopotamia, the fertile plain between the Euphrates and Tigris Rivers in what is now Iraq. We are fortunate that the peoples of this region kept extensive records of their society — and their mathematics — on hardy sun-baked clay tablets. A large number of these tablets survive today. The Babylonians used a simple stylus to make marks in the clay and developed a form of writing based on cuneiform (wedge-shaped) symbols.

The mathematical activity of the Babylonians seems to have been motivated, at first, by the practical everyday needs of running their society. Many problems described in early tablets are concerned with calculating the number of workers needed for building irrigation canals and the total expense of wages, for instance. But many problems described in later texts have no apparent practical application and clearly indicate an interest in pursuing mathematics for its own sake.

The Babylonians used only two symbols to represent numbers: the symbol to represent a unit and the symbol to represent a group of ten. A simple additive system was used to represent the numbers 1 through 59. A base-60 place-value system was then used to represent numbers greater than 59. Spaces were inserted between clusters of symbols.

Historians are not clear as to why the Babylonians chose to work with a sexagesimal system. A popular theory suggests that this number system is based on the observation that there are 365 days in the year. When rounded to the more convenient (highly divisible) value of 360, we have a multiple of 60. Vestiges of this number system remain with us today. For example, we use the number 360 for the number of degrees in a circle, and we count 60 seconds in a minute and 60 minutes per hour.

There were two points of possible confusion with the Babylonian numeral system. With no symbol for zero, it is not clear whether the numeral represents 61 (as one unit of 60 plus a single unit), 3601 (as one unit of 602 plus a single unit), or even 216,060, for instance. Also, the Babylonians were comfortable with fractions and used negative powers of 60 to represent them (just as we use negative powers of 10 to write fractions in decimal notation). But with no notation for the equivalent of a decimal point, the symbol could also be interpreted to mean 1 + (1/60), or (1/60) + (1/602), or even 60 + (1/604), for instance. As the Babylonians never developed a method for resolving such ambiguity, we assume then that it was never considered a problem for scholars of the time. (Historians suggest that the context of the text always made the interpretation of the numeral apparent.)

The Babylonians compiled extensive tables of powers of numbers and their reciprocals, which they used in ingenious ways to perform arithmetic computations. (For instance, a tablet dated from 2000 B.C.E. lists all the squares of the numbers from one to 59, and all the cubes of the numbers from one to 32.) To compute the product of two numbers “a” and “b”, Babylonian scholars first computed their sum and their difference, read the squares of those numbers from a table, and divided their difference by four. (In modern notation, this corresponds to the computation: $ab = (1/4)\ [(a + b)^2 - (a - b)^2]$.) To divide a number "a" by "b", scholars computed the product of "a" and the reciprocal 1/b (recorded in a table): $ab = a \times (1/b)$. The same table of reciprocals also provided the means to solve linear equations: $bx = a$. (Multiply "a" by the reciprocal of "b".)

Problems in geometry and the computation of area often lead to the need to solve quadratic equations. For instance, a problem from one tablet asks for the width of a rectangle whose area is 60 and whose length is seven units longer than the width. In modern notation, this amounts to solving the equation $x(x + 7) = x^2 + 7x = 60$. The scribe who wrote the tablet then proffers a solution that is equivalent to the famous quadratic formula: $x = \sqrt{(7/2)^2 + 60} - (7/2) = 5$. (Square roots were computed by examining a table of squares.)

Problems about volume lead to cubic equations, and the Babylonians were adept at solving special equations of the form: $ax^3 + bx^2 = c$. (They solved these by setting $n = (ax)/b$, from which the equation can be rewritten as $n^3 + n^2 = ca^2/b^3$. By examining a table of values for $n^3 + n^2$, the solution can be deduced.)

It is clear that Babylonian scholars knew of Pythagoras's theorem, although they wrote no general proof of the result. If the width of a rectangle is four units and the length of its diagonal is five units, what is its breadth? Four times four is 16, and five times five is 25. Subtract 16 from 25 and there remains nine. What times what equals nine? Three times three is nine. The breadth is three.

The Babylonians used Pythagoras's theorem to compute the diagonal length of a square, and they found an approximation to the square root of two accurate to five decimal places. (It is believed that they used a method analogous to Heron's method to do this.) Babylonian scholars were also interested in approximating the areas and volumes of various common shapes by using techniques that often invoked Pythagoras's theorem.

Most remarkable is a tablet that lists 15 large Pythagorean triples. As there is no apparent practical need to list these triples, this strongly suggests that the Babylonians did indeed enjoy mathematics for its own sake.

(by James Tanton, from Encyclopedia of Mathematics)