Activity 179. Read the article. Review your answers to the quiz in Activity 178.

The ancient Greeks of ca. 600 B.C.E. to ca. 480 C.E. set the current standards of logical rigour in mathematics. Although many ancient cultures practiced and developed mathematics, it was the Greeks who developed the explicit art of "proof" and explored the power of pure deductive reasoning to its fullest.

We should mention that when speaking of "Greek mathematics," historians include any mathematician who wrote in the Greek language and followed the Greek tradition of mathematical thought. Greek was the common language of the Mediterranean world during ancient times, and many intellectuals from different parts of that region are today considered Greek scholars. For instance, the great Archimedes was from Syracuse, now a part of Italy, and Euclid (ca. 300–260 B.C.E.) is believed to have lived in Alexandria, Egypt.

There are very few original records of Greek work. Initially, knowledge was transmitted only orally from teacher to student. Around 450 B.C.E. the Greeks adopted the ancient Egyptian practice of writing on papyrus scrolls. Unfortunately, papyrus — a grasslike plant grown in the Nile Delta region — decays rapidly away from the exceptionally dry climate of Egypt. The Greeks combated this problem by repeatedly making copies of their works but, because of the effort involved, copied only those pieces they deemed of utmost importance. The first mathematical work preserved and honoured this way was Euclid's masterpiece "The Elements" of ca. 300 B.C.E. Historians have had to rely on commentary made by later scholars to deduce what was accomplished mathematically before the time of Euclid.

Greek scholars approached all of mathematics through the study of geometry. Even their work on the properties of whole numbers, ratios, and proportions, as well as mechanics and astronomy was done in a geometric style. A "number," for instance, was literally a line segment, and a "ratio" was understood in terms of commensurable segments. It is interesting to note that Greek scholars took careful steps to avoid speaking directly of the infinite. (The 5^{th}-century B.C.E. paradoxes on the nature of motion and the infinitely small developed by Zeno of Elea deeply affected Greek thinking.) For instance, Euclid stated that any line segment could be extended to any arbitrary length, but never spoke of lines that were infinitely long. In Euclid's proof of the infinitude of primes, Euclid stated that from any finite list of prime numbers one can always construct one more, but never spoke of the set of primes as infinite.

Many historians regard Thales of Miletus (ca. 625–547 B.C.E.) as the first Greek mathematician of note. Commentaries suggest that Thales identified, and proved, seven key geometric propositions, including that the base angles of an isosceles triangle are always equal and that the inscribed angle from the diameter of a circle is always a right angle, for instance. The great scholar and mystic Pythagoras lived a century later, and he and his followers are credited with the discovery of the famous result about right triangles (today called Pythagoras's theorem) and the discovery of irrational numbers. A great deal of

mystery surrounds the life and legend of Pythagoras. He founded a semireligious sect called the Pythagorean Brotherhood (women were equal members) based on certain mystic significances ascribed to whole numbers and their ratios.

The great philosopher Plato (428–348 B.C.E.) wrote a great deal about mathematics in his famous dialogues, demonstrating a deep personal respect for the subject. The five regular polyhedra — the Platonic solids — are named in his honour. In his philosophical treatises, Plato used the example of mathematics as something that cannot be discovered by the senses but can nonetheless be discovered by the power of logical reasoning. He also believed mathematics to be an essential part of a cultured person's education. Philosopher Aristotle (384–322 B.C.E.) adopted the same view and used mathematics as examples in his development of formal logic and his analysis of arguments.

Today, the Greek scholar Euclid is considered to be the most influential mathematics scholar of all time. In his famous work "The Elements", Euclid collated all mathematical knowledge known at his time into a single tome. Although an impressive feat, it was the organization of the text that had the greatest impact. Beginning with a small collection of "self-evident truths," Euclid showed that all mathematical knowledge of his time could be deduced by pure logical reasoning alone. This work demonstrated the power of the mind and set the model for all mathematical research in the future. Mathematicians today still work to the standards of rigour as set by Euclid. Next to the Bible, Euclid's "The Elements" is the most widely published book of all time.

After producing "The Elements", Euclid continued work on the conic sections, on optics, and on general problems in geometry. He continued interest in constructible numbers and no doubt contemplated the classic Greek problem of squaring the circle. (In "The Elements" Euclid had demonstrated general procedures for squaring arbitrary polygonal figures.) This challenge, as well as the problems of trisecting an angle and duplicating the cube, spurred a great deal of significant further research in mathematics for centuries to come.

Archimedes of Syracuse (ca. 287–212 B.C.E.) solved the problem of squaring the parabola, as well as made significant advances in computing the areas and volumes of curved figures and solids. (He also "solved" the problem of squaring the circle by making use of his Archimedean spiral. Unfortunately, his method went beyond the use of a straightedge and compass alone, and so is not a permissible solution to the original problem.)

Apollonius of Perga (ca. 262–190 B.C.E.) continued work on conic sections and is credited for properly defining an ellipse, a hyperbola, and a parabola. Around the same time, Greek astronomer Hipparchus wrote a table of "chord values" (the equivalent to a modern table of sine values), which he used to solve astronomical problems. This represented the beginning development of trigonometry in Greek mathematics, but also marked an end of fervent mathematical development in the Greek tradition. For the five centuries that

followed, new developments were limited to straightforward advances in astronomy, trigonometry, and algebra, with just a few notable exceptions.

In the 2nd century C.E., The Greek astronomer Claudius Ptolemy corrected and extended Hipparchus's table and clarified the mathematics that is used to produce such a table. He is also known as one of the first scholars to make a serious attempt at proving Euclid's parallel postulate. In the 3rd century, Diophantus of Alexandria produced his famous text "Arithmetic", from which the study of Diophantine equations was born. In the mid-4th century, the enthusiastic Pappus of Alexandria attempted to revive interest in ardent mathematical research of the Greek style. He produced his treatise "Synagoge" (Collection) to act as a commentary and guide to all the geometric works of his time and included in it a significant number of original results, extensions of ideas, and innovative shifts of perspective. Unfortunately, he did not succeed in his general goal. After Pappus, of note is Hypatia of Alexandria (370–415), the first woman to be named in the history of mathematics, credited for writing insightful commentaries on the works of Apollonius and Diophantus, and Proclus (ca. 410–485), who is noted for his detailed commentary on the work of Euclid and his own attempt to prove the parallel postulate.

The beginning of the 5th century marks a clear end to the tradition of Greek mathematics.

(by James Tanton, from Encyclopedia of Mathematics)