

**Activity 186. Read the article. Review your answers to the quiz in Activity 185.**

The entire course of Western mathematics was profoundly affected by a single Indian invention, that of the place-value decimal system. That every possible number can be expressed via a set of just 10 symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, by making careful use of the place of each symbol, seems such a simple notion nowadays that it is hard to appreciate its profound importance. This elegant notation system provided the means for Indian scholars to perform complicated arithmetical computations with relative ease, which in turn led to significant developments in numerical techniques, approximation methods, and the theory of arithmetic. Only when other cultures adopted the place-value decimal system from India could they accomplish the same mathematical feats that this culture had already developed.

The earliest dated evidence of mathematical activity in the Indian subcontinent goes back to the Indus civilization of 2500 B.C.E. Bronze weights and graded rods (rulers) from the period show that these people were already working with a decimal system. The Indus people worked with a basic unit of length 1.32 in. long (today called the “Indus inch”), 10 of which make their version of a “foot.” Excavations show that the weights and graded rods were used extensively in construction.

The earliest written records of Indian culture are the religious texts the Vedas, dating between 1500 B.C.E. and 800 B.C.E. Although not mathematical in content, appendices to the texts give specific rules for constructing altars, exhibiting a thorough understanding of the basic principles of geometry. Early versions of the digits 0 through 9 were used at this time.

By 600 C.E., the Vedic religion had gone into decline, and Jainism came to the fore. During this period mathematics was driven by the needs of the religion and its demands for careful astronomical observations to pinpoint the exact times of religious observances and the development of an accurate calendar. The decimal representation system was now fully developed, and scholars were able to make precise and surprisingly accurate calculations. The mathematician Aryabhata (ca. 500 C.E.), for instance, had developed a theory of trigonometry to aid astronomical calculations, had developed methods for extracting square roots, evaluated  $\pi$  to a high degree of accuracy, and was able to find integer solutions to a large class of equations that arose from astronomical theories.

One written text from this period was discovered in 1881 in the town of Bakhshali, now in Pakistan. Written on birch bark, the Bakhshali manuscript shows that mathematicians were also comfortable with fractions, basic algebraic manipulations (they used a dot to represent an unknown quantity), and sophisticated approximation formulae.

Two mathematical research centres were formed in India during the era of Jainism, both astronomical observatories. Aryabhata headed the first centre at Kusumapura in the northeast of the Indian subcontinent, and the mathematician Varahamihira, who also made

contributions to astronomy and trigonometry, headed the second centre at Ujjain, also in the north.

Varahamihira was succeeded by the 7<sup>th</sup>-century mathematician Brahmagupta, who, in his famous work “The Opening of the Universe”, introduced and explained the arithmetic of non-positive numbers. He was the first mathematician in history to give zero the status of a number, defining it to be the result of subtracting a quantity from itself. Brahmagupta’s work also includes a formula for the area of a cyclic quadrilateral in terms of its sides (today called Brahmagupta’s formula), and presents methods for solving linear and quadratic equations, as well as systems of equations. Brahmagupta also developed sophisticated interpolation techniques for computing sine values in trigonometry.

For the next 200 years, Indian scholars worked to refine further methods of trigonometry and techniques of astronomical calculation. The mathematician Bhaskara II (Bhaskaracharya) of the 12<sup>th</sup> century made advances in number theory, algebra, combinatorics, and astronomy, and wrote a comprehensive text summarizing the state of mathematics and astronomy in India at his time. Soon afterward, other Indian scholars developed these ideas further. Jaina mathematicians also clarified the standard exponent rules and manipulated exponents in a manner that suggests today that they were also familiar with the basic principles of logarithms.

The 14<sup>th</sup>-century scholar Madhava of Sangamagramma made significant advances in analysis. He produced the infinite series expansions of trigonometric and inverse trigonometric functions (today called Taylor series), discovered the binomial theorem, and even produced Gregory’s series for  $\pi$ , which he used to approximate its value to a considerably accurate degree.

During the first millennium India had very little contact with the cultures of the West. News of the decimal representation system, however, did manage to spread to other countries relatively quickly. A manuscript written in Syria in 662 discusses the new method of calculation, and there is evidence that the decimal system was being used in Cambodia and other Asian countries soon afterward. By the 9<sup>th</sup> century, the decimal system was in common use in the Islamic world, and from there it was quickly transmitted to Europe. Arab scholars maintained a keen interest in the work of Indian mathematicians for the centuries that followed and took an active role in preserving and translating many Indian texts.

*(by James Tanton, from Encyclopedia of Mathematics)*