Activity 229. Read the extract from the 2003 novel “The Housekeeper and the Professor” by Japanese writer Yoko Ogawa. Explain the concept of “amicable numbers” based on the text.

"Do you send a lot of articles to magazines?" I asked.

"I wouldn't call them 'articles.' They're just puzzles for amateur mathematicians. Sometimes there's even a prize. Wealthy men who love mathematics put up the money." He looked down, checking his suit in various places, and his gaze fell on a note clipped to his left pocket. "Oh, I see. I sent a proof to the Journal of Mathematics today."

It had been much more than eighty minutes since I'd made my trip to the post office.

"Oh, dear!" I said. "If it's a contest, I should have sent it express mail. If it doesn't get there first, I suppose you don't get the prize."

"No, there was no need to send it express. Of course, it's important to arrive at the correct answer before anyone else, but it's just as important that the proof is elegant."

"I had no idea a proof could be beautiful... or ugly."

"Of course, it can," he said. Getting up from the table, he came over to the sink where I was washing the dishes and peered at me as he continued. "The truly correct proof is one that strikes a harmonious balance between strength and flexibility. There are plenty of proofs that are technically correct but are messy and inelegant or counterintuitive. But it's not something you can put into words — explaining why a formula is beautiful is like trying to explain why the stars are beautiful."

I stopped washing and nodded, not wanting to interrupt the Professor's first real attempt at conversation.

"Your birthday is February twentieth. Two twenty. Can I show you something? This was a prize I won for my thesis on transcendent number theory when I was at college." He took off his wristwatch and held it up for me to see. It was a stylish foreign brand, quite out of keeping with the Professor's rumpled appearance.

"It's a wonderful prize," I said.

"But can you see the number engraved here?" The inscription on the back of the case read President's Prize No. 284.

"Does that mean that it was the two hundred and eighty-fourth prize awarded?"

"I suppose so, but the interesting part is the number 284 itself. Take a break from the dishes for a moment and think about these two numbers: 220 and 284. Do they mean anything to you?"

Pulling me by my apron strings, he sat me down at the table and produced a pencil stub from his pocket. On the back of an advertising insert, he wrote the two numbers, separated strangely on the card.

220

284

"Well, what do you make of them?"

I wiped my hands on my apron, feeling awkward, as the Professor looked at me expectantly. I wanted to respond but had no idea what sort of answer would please a mathematician. To me, they were just numbers.

"Well...," I stammered. "I suppose you could say they're both three-digit numbers. And that they're fairly similar in size — for example, if I were in the meat section at the supermarket, there'd be very little difference between a package of sausage that weighed 220 grams and one that weighed 284 grams. They're so close that I would just buy the one that was fresher. They seem pretty much the same — they're both in the two hundreds, and they're both even —"

"Good!" he almost shouted, shaking the leather strap of his watch. I didn't know what to say. "It's important to use your intuition. You swoop down on the numbers, like a kingfisher catching the glint of sunlight on the fish's fin." He pulled up a chair, as if wanting to be closer to the numbers. The musty paper smell from the study clung to the Professor.

"You know what a factor is, don't you?"

"I think so. I'm sure I learned about them at some point..."

"For 220 is divisible by 1 and by 220 itself, with nothing leftover. So, 1 and 220 are factors of 220. Natural numbers always have 1 and the number itself as factors. But what else can you divide it by?"

"By 2, and 10..."

"Exactly! So, let's try writing out the factors of 220 and 284, excluding the numbers themselves. Like this."

220 : 1 2 4 5 10 11 20 22 44 55 110

142 71 4 2 1 : 284

The Professor's figures, rounded and slanting slightly to one side, were surrounded by black smears where the pencil had smudged.

"Did you figure out all the factors in your head?" I asked.

"I don't have to calculate them — they just come to me from the same kind of intuition you used. So then, let's move on to the next step," he said, adding symbols to the lists of factors.

220 : 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 =

= 142 + 71 + 4 + 2 + 1 : 284

"Add them up," he said. "Take your time. There's no hurry."

He handed me the pencil, and I did the calculation in the space that was left on the advertisement. His tone was kind and full of expectation, and it didn't seem as though he were testing me. On the contrary, he made me feel as though I were on an important mission, that I was the only one who could lead us out of this puzzle and find the correct answer.

I checked my calculations three times to be sure I hadn't made a mistake. At some point, while we'd been talking, the sun had set, and night was falling. From time to time, I heard water dripping from the dishes I had left in the sink. The Professor stood close by, watching me.

"There," I said. "I'm done."

220 : 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284

220 = 142 + 71 + 4 + 2 + 1 : 284

"That's right! The sum of the factors of 220 is 284, and the sum of the factors of 284 is 220. They're called 'amicable numbers,' and they're extremely rare. Fermat and Descartes were only able to find one pair each. They're linked to each other by some divine scheme, and how incredible that your birthday and this number on my watch should be just such a pair."

We sat staring at the advertisement for a long time. With my finger I traced the trail of numbers from the ones the Professor had written to the ones I'd added, and they all seemed to flow together, as if we'd been connecting up the constellations in the night sky.

(from "The Housekeeper and the Professor,"

written by Yoko Ogawa, translated by Stephen Snyder,

2003)