

Activity 62. Read the article to outline the system of Roman numerals and that of Hindu-Arabic numerals.

More than 5,000 years ago an Egyptian ruler recorded, perhaps with a bit of exaggeration, the capture of 120,000 prisoners, 400,000 oxen, and 1,422,000 goats. This event was inscribed on a ceremonial mace which is now in a museum in Oxford, England.

The ancient Egyptians developed the art of counting to a high degree, but their system of numeration was very crude. For example, the number 1,000 was symbolized by a picture of a lotus flower and the number 2,000 was symbolized by a picture of two lotus flowers growing out of a bush. Although these symbols, called hieroglyphics, permitted the Egyptians to write large numbers, the numeration system was clumsy and awkward to work with. The number 999, for instance, required 27 individual marks.

In our system of numeration, we use ten symbols called digits — 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 — and combinations of these symbols. Our system of numeration is called decimal, or base-ten, system. There is little doubt that our ten fingers influenced the development of a numeration system based on ten digits.

The ancient Hindus are credited with discovering the decimal system of numeration we use today. This system was translated into Arabic prior to its introduction into Europe by travelling merchants around the 13th century. Hence it is also known as the Hindu-Arabic system. Adoption of the Hindu-Arabic system met resistance due to the widespread use of the Roman numeral system during this period.

Based on a simple tally system similar to the one used by the ancient Egyptians, merchants of the Roman empire of about 500 B.C.E. used letter symbols for powers of 10 and for the intermediate values of 5, and simply grouped symbols together to represent all other quantities.

The expression CLXXIII, for instance, represented the number $100 + 50 + 10 + 10 + 1 + 1 + 1 = 173$. Although the order of the symbols was not important, it became the convention to list symbols from largest to smallest, left to right.

Initially the symbols D and M were not part of the Roman system. The number 1,000 was written (I), and further applications of round brackets allowed for the expression of even greater quantities. For instance, ((I)) represented 10,000, and (((I))) represented 100,000. Stonemasons introduced the symbols D and M to simplify their work.

The Romans also introduced other ornamentations to increase the value of a numerical symbol. For instance, vertical bars were used to represent a 100-fold increase, and a bar placed above the symbol represented a 1,000-fold increase.

There was no symbol for zero in the Roman system. To avoid the four-fold repetition of symbols (as in the expression CCCCLXXXIIII for 444), a subtractive principle was introduced in the 13th century:

The placement of a small value immediately to the left of a higher value indicates that that small value is to be subtracted from the higher value.

Thus, 4 could be written as IV, 90 as XC, and 444 as CDXLIV. The subtractive principle was subject to two rules:

- 1. The symbols V, L, and D cannot be used as the numbers to be subtracted.*
- 2. Only one symbol I, X, or C can be placed before a higher number symbol.*

Thus, for example, it was not permissible to write IIX for eight. Although not a proper place-value system, with the subtractive principle in use, the order of the symbols used was now important.

Performing operations of basic arithmetic with Roman numerals is very awkward. For example, it is not immediate what the solution to the following addition problem should be:

$$XLIV + XVII + XXIX$$

That European merchants were comfortable working with the Roman numeral system for well over a millennium suggests that scholars did not use the numeral system to perform calculations, only to record the results. (Arithmetic was performed on a counting board such as an abacus.)

The system of Roman numerals remained popular in Western Europe until the 17th century. Although the system was eventually replaced by the Hindu-Arabic numeral system we use today, it still remains a tradition to use Roman numerals for numbering introductory pages in books, for instance.

Using a base-10 place-value system, numbers in the Hindu-Arabic system are expressed via combinations of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, organized so as to represent groupings of powers of 10. For instance, the number 574 represents the five groups of 100, seven groups of 10, and four single units.

This numerical system originated from India around 600 C.E., almost in the exact same form as we use it today. The system was transmitted to the Arabs two centuries later as they worked to translate the Sanskrit works on astronomy into Arabic. The Arab mathematician Muhammad ibn Musa al-Khwarizmi (ca. 800) wrote an influential treatise describing the Hindu numeral system and used it in his famous book “Calculation by Restoration and Reduction”, from whose title the modern word “algebra” is derived. As Western scholars began translating the Arabic texts into Latin, word of the efficient numeration system spread across Western Europe. The Italian scholar Fibonacci (ca. 1170–1250) avidly promoted their use. By the end of the 17th century, the Hindu-Arabic numeral system completely replaced the cumbersome system of Roman numerals that were the standard in Europe for over 1,500 years.

Other numeration systems were developed in early cultures and societies. Two of the most common were the base-five system, related to the number of fingers on one hand and the base-twenty system, related to the number of fingers and toes.

In some languages the word for “five” is the same as the word for “hand”, and the word for “ten” is the same as the word for “two hands”. In the English language the word “digit” is a synonym for the word “finger” — that is, ten digits, ten fingers.

Still another early system of numeration was a base-sixty system developed by the Mesopotamians and used for centuries. These ancient people divided the years into 360 days (6×60); today we still divide the hour into 60 minutes and the minute to 60 seconds. Numeration systems of current interest include a binary, or base-two, system used in electronic computers and a base-twelve, or duodecimal system.