Activity 76. Read the article. Expand on what constitutes the scope of modern algebra as a distinct branch of mathematics.

The branch of mathematics concerned with the general properties of numbers, and generalizations arising from those properties, is called algebra. Often symbols are used to represent generic numbers, thereby distinguishing the topic from the study of arithmetic. For instance, the equation $2 \times (5 + 7) = 2 \times 5 + 2 \times 7$ is a (true) arithmetical statement about a specific set of numbers, whereas the equation $x \times (y + z) = x \times y + x \times z$ is a general statement describing a property satisfied by any three numbers. It is a statement in algebra.

Much of elementary algebra consists of methods of manipulating equations to either put them in a more convenient form, or to determine (that is, solve for) permissible values of the variables that appear. For instance, rewriting $x^2 + 6x + 9 = 25$ as $(x + 3)^2 = 25$ allows an easy solution for "x": either $x + 3 = 5$, yielding $x = 2$, or $x + 3 = -5$, yielding $x = -8$.

The word "algebra" comes from the Arabic term "al-jabr" used by the great Muhammad ibn Musa al-Khwarizmi (ca. 780–850) in his writings on the topic.

In modern times the subject of algebra has been widened to include abstract algebra, group theory, and the study of alternative number systems such as modular arithmetic. Boolean algebra looks at the algebra of logical inferences, matrix algebra the arithmetic of matrix operations, and vector algebra the mechanics of vector operations and vector spaces.

An algebraic structure is any set equipped with one or more operations (usually binary operations) satisfying a list of specified rules. For example, any group, ring, field, or vector space is an algebraic structure.

Research in pure mathematics is motivated by one fundamental question: what makes mathematics work the way it does? For example, to a mathematician, the question, "What is 263 × 178 (or equivalently, 178 × 263)?" is of little interest. A far more important question would be, "Why should the answers to 263 × 178 and 178 × 263 be the same?"

The topic of abstract algebra attempts to identify the key features that make algebra and arithmetic work the way they do. For example, mathematicians have shown that the operation of addition satisfies five basic principles, and that all other results about the nature of addition follow from these.

1. *Closure: The sum of two numbers is again a number.*
2. *Associativity: For all numbers "a", "b", and "c", we have: $(a + b) + c = a + (b + c)$.*
3. *Zero element: There is a number, denoted "0," so that: $a + 0 = a = 0 + a$ for all numbers "a".*
4. *Inverse: For each number "a" there is another number, denoted "–a," so that: $a + (-a) = 0 = (-a) + a$.*
5. *Commutativity: For all numbers "a" and "b" we have: $a + b = b + a$.*

Having identified these five properties, mathematicians search for other mathematical systems that may satisfy the same five relations. Any fact that is known about addition will consequently hold true in the new system as well. This is a powerful approach to matters. It avoids having to re-prove theorems and facts about a new system if one can recognize it as a familiar one in disguise. For example, multiplication essentially satisfies the same five axioms as above, and so for any fact about addition, there is a corresponding fact about multiplication. The set of symmetries of a geometric figure also satisfy these five axioms, and so too all known results about addition immediately transfer to interesting statements about geometry. Any system that satisfies these basic five axioms is called an "Abelian group," or just a group if the fifth axiom fails. Group theory is the study of all the results that follow from these basic five axioms without reference to a particular mathematical system.

The study of rings and fields considers mathematical systems that permit two fundamental operations (typically called addition and multiplication). Allowing for the additional operation of scalar multiplication leads to a study of vector spaces.

The theory of algebraic structures is highly developed. The study of vector spaces as well as matrices, for example, is so extensive that the topic is regarded as a field of mathematics in its own right and is called linear algebra. As matrices are used to analyze and solve systems of simultaneous linear equations and to describe linear transformations between vector spaces, this topic of study unites geometric thinking with numerical analysis. As the set of all invertible matrices of a given size form a group, called the general linear group, techniques of abstract algebra can also be incorporated into this work.

(from Elementary Algebra)