

### **Activity 91. Divide the text into paragraphs.**

A straight line, usually horizontal, for which each point on the line represents a real number  $R$  is called a number line (real line). One assumes that the line extends indefinitely both to the left and to the right. A single point  $O$  on the line, called the origin, corresponds to the number zero in the real number system, and it is conventional to assume that a point “ $a$ ” distance “ $a$ ” units to the right of  $O$  represents the positive real number “ $a$ ” and a point “ $b$ ” units to the left of  $O$  the negative real number  $-b$ . The integers are thus represented as evenly spaced points, one unit apart, along the line. A number line is a one-dimensional Cartesian coordinate system (rectangular coordinate system, orthogonal coordinate system). The theory of cardinality shows that there are just as many points on the number line as there are points in a two-dimensional plane. The diagonal argument shows that the set of rational numbers (fractions) take up absolutely no space on the number line. A set of numbers used to locate a point on a number line, in a plane, or in space are called the coordinates of that point. For example, the coordinates of points on a number line could be given by their distances from a fixed point  $O$ , with points on one specified side of  $O$  being deemed a positive distance from  $O$ , and the points on the opposite side of  $O$  a negative distance from  $O$ . One way of assigning coordinates to points in the plane is to establish a fixed point  $O$  in the plane, and two lines of reference (called axes) that pass through  $O$ . Each axis is divided into a positive side and a negative side by  $O$ . Given a point  $P$  in the plane, one draws lines through  $P$  parallel to each of the axes. The distances along which these new lines intersect the axes specify the location of the point  $P$ . When the axes are drawn at right angles, the system is called a Cartesian coordinate system. The axes are usually called the  $x$ - and  $y$ -axes, and the pair of numbers  $(x,y)$  specifying the location of a point  $P$  (as “ $x$ ” units along one axis, and “ $y$ ” units along the second) are called the Cartesian coordinates of  $P$ . In three-dimensional space, the location of points can be specified via three mutually perpendicular (or oblique) axes passing through a common point  $O$ . The idea of assigning sets of numbers to points to specify locations is an old one. By the 3<sup>rd</sup> century B.C.E., Greek scholars Apollonius of Perga and Archimedes of Syracuse had used longitude, latitude, and altitude to define the position of a point on the Earth’s surface. Roman and Greek surveyors labelled maps with grid lines, so as to specify locations via row and column numbers. One of the biggest breakthroughs in the development of mathematics occurred when geometry and algebra were united through the invention of the Cartesian coordinate system. Credited to 17<sup>th</sup>-century French mathematician and philosopher René Descartes (whose name Latinized reads Cartesius), Cartesian coordinates (rectangular coordinates, orthogonal coordinates) provide a means of representing each point in the plane via a pair of numbers. One begins by selecting a fixed point  $O$  in the plane, called the origin, and drawing through it two perpendicular number lines, called axes, one horizontal and one vertical, and both with the point  $O$  at the zero position on the line. It has become the convention to set the positive side of the horizontal number line to the right of  $O$ , and the positive side of the vertical number line above  $O$ , and to call the horizontal axis the  $x$ -axis, and the vertical one the  $y$ -axis. The Cartesian coordinates of a point  $P$  in the plane is a pair of numbers  $(x,y)$  which

then describes the location of that point as follows: *The x-coordinate, or “abscissa,” is the horizontal distance of the point from O along the horizontal axis. (A positive distance represents a point to the right of the vertical axis; a negative distance one to the left.) The y-coordinate, or “ordinate,” is the vertical distance of the point from O along the vertical axis. (A positive distance represents a point located above the horizontal axis, and a negative distance one located below.)* Extending this idea to three-dimensions, points in space can be specified by a triple of numbers  $(x,y,z)$  representing the distances along three mutually perpendicular number lines. The coordinate axes are then called the x-, y-, and z-axes. They intersect at a point O, which is zero on all three number lines. The axes could be oriented to either form a left-handed or a right-handed system. The advent of a coordinate system allowed mathematicians, for the first time, to bring the power of algebra to the study of geometry. The French mathematician Nicole Oresme (1323–82) was the first to describe a way of graphing the relationship between an independent variable and a dependent one, and thus the first to make steps toward uniting geometry and algebra. The explicit construction of what we would call a coordinate system first appeared with the work of the French lawyer and amateur mathematician Pierre de Fermat (1601–65). Starting with some horizontal reference line to represent an independent variable “x”, Fermat would graphically depict the relationship of a second variable “y” to it as a line segment, held at a fixed angle to the reference line, whose length would vary according to the variable “y” as it slides along the x-axis. Fermat did not think in terms, however, of identifying a second axis, nor did he require the line segment representing “y” to be perpendicular to the x-axis. In his famous 1637 text “Geometry”, René Descartes independently described similar methods for representing algebraic relationships graphically. Because the work of Fermat was not published until after his death, the discovery of coordinate geometry was attributed to Descartes. Because Fermat and Descartes interpreted the unknown variable “y” in an algebraic relationship as a physical length, both scholars only ever considered positive coordinates. The English mathematician John Wallis (1616–1703) was the first to introduce the possibility of negative coordinates. The idea of setting a fixed second axis, the y-axis, perpendicular to the x-axis was not popular until the mid 1700s. It was an idea that seemed to evolve gradually.

*(from Elementary Geometry for College Students)*